

Deep Generative Models: GANs and VAE

Jakub M. Tomczak
AMLAB, Universiteit van Amsterdam

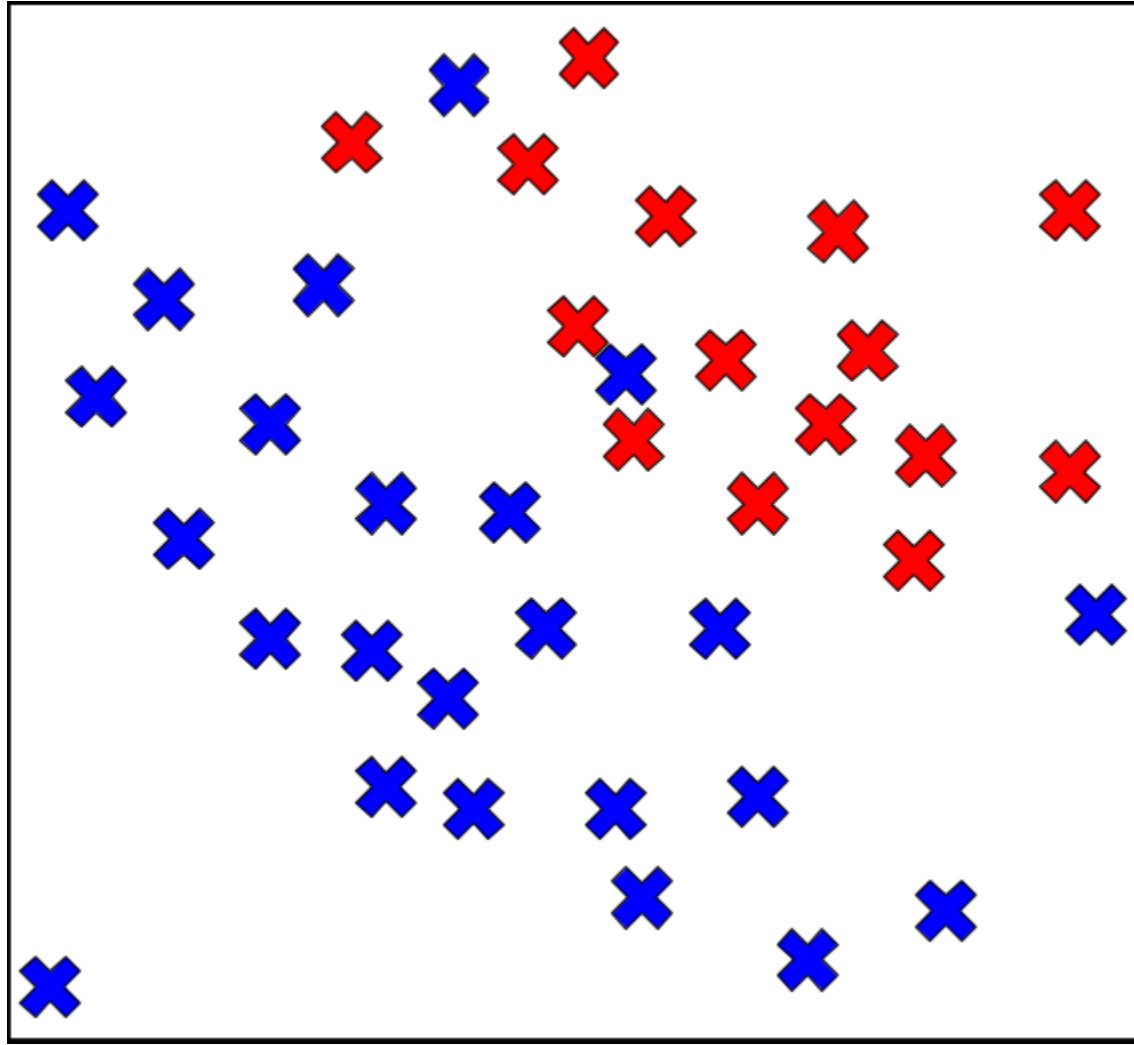
Split, Croatia 2017



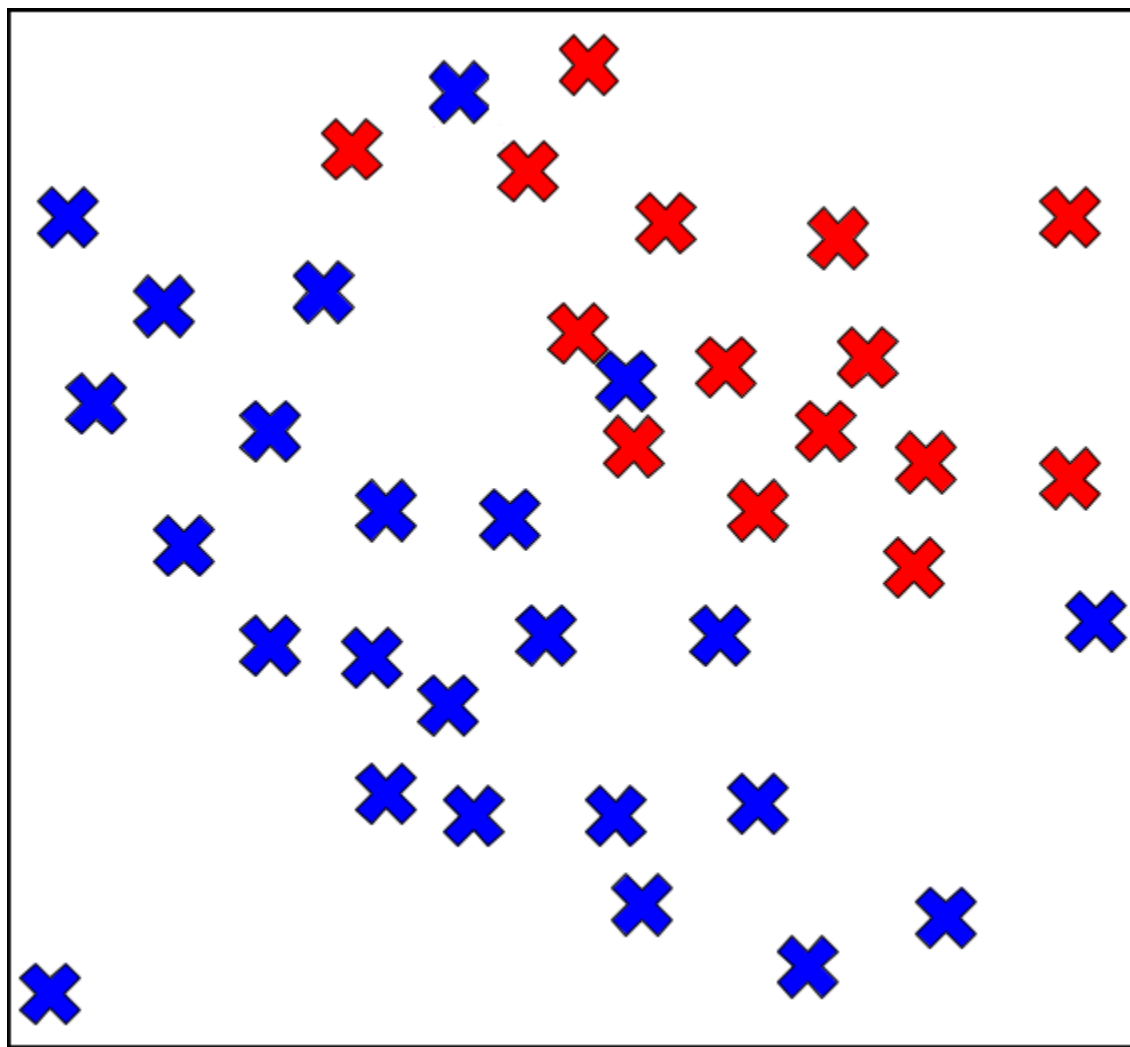
UNIVERSITY OF AMSTERDAM

Do we need generative modeling?

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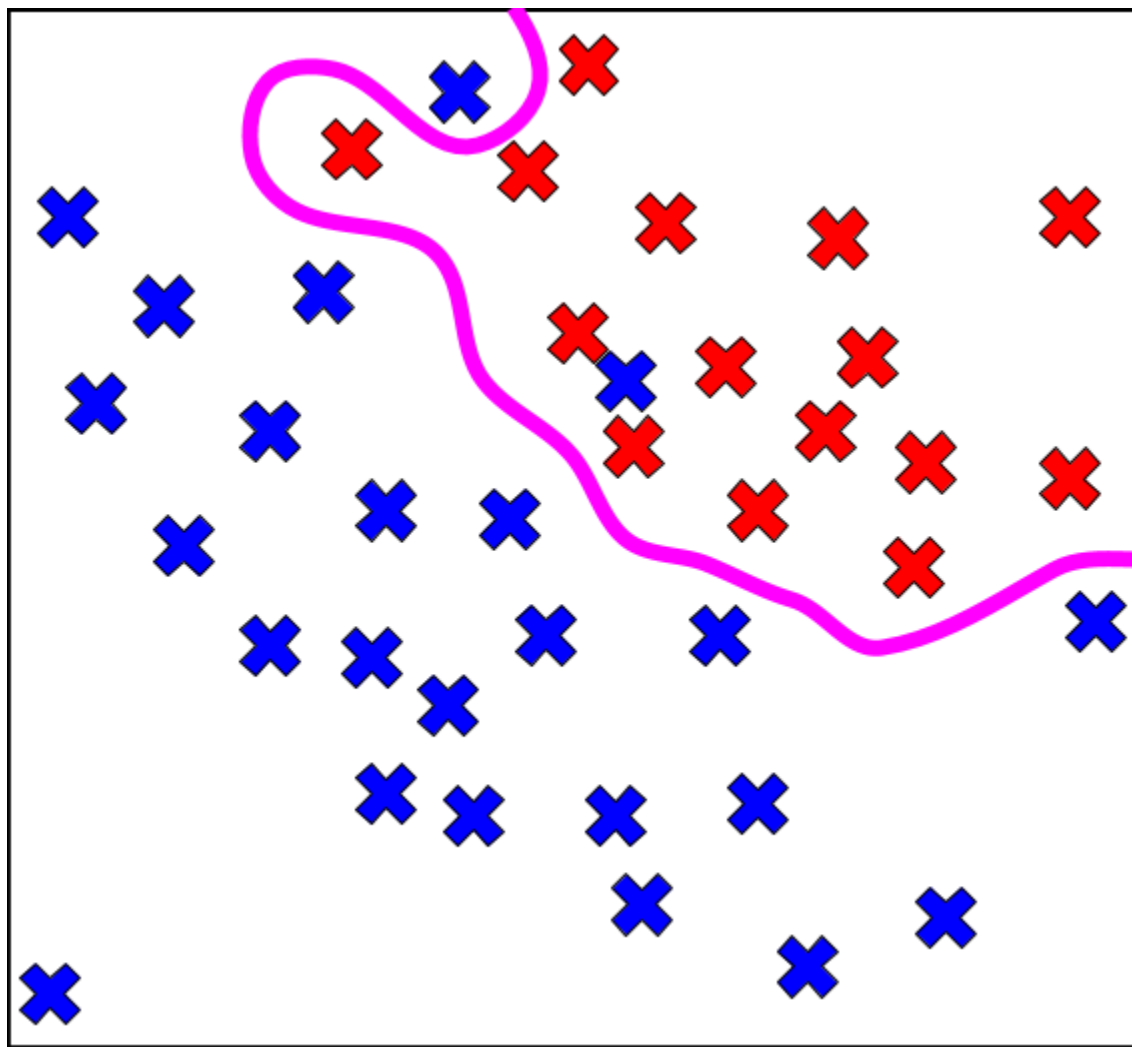


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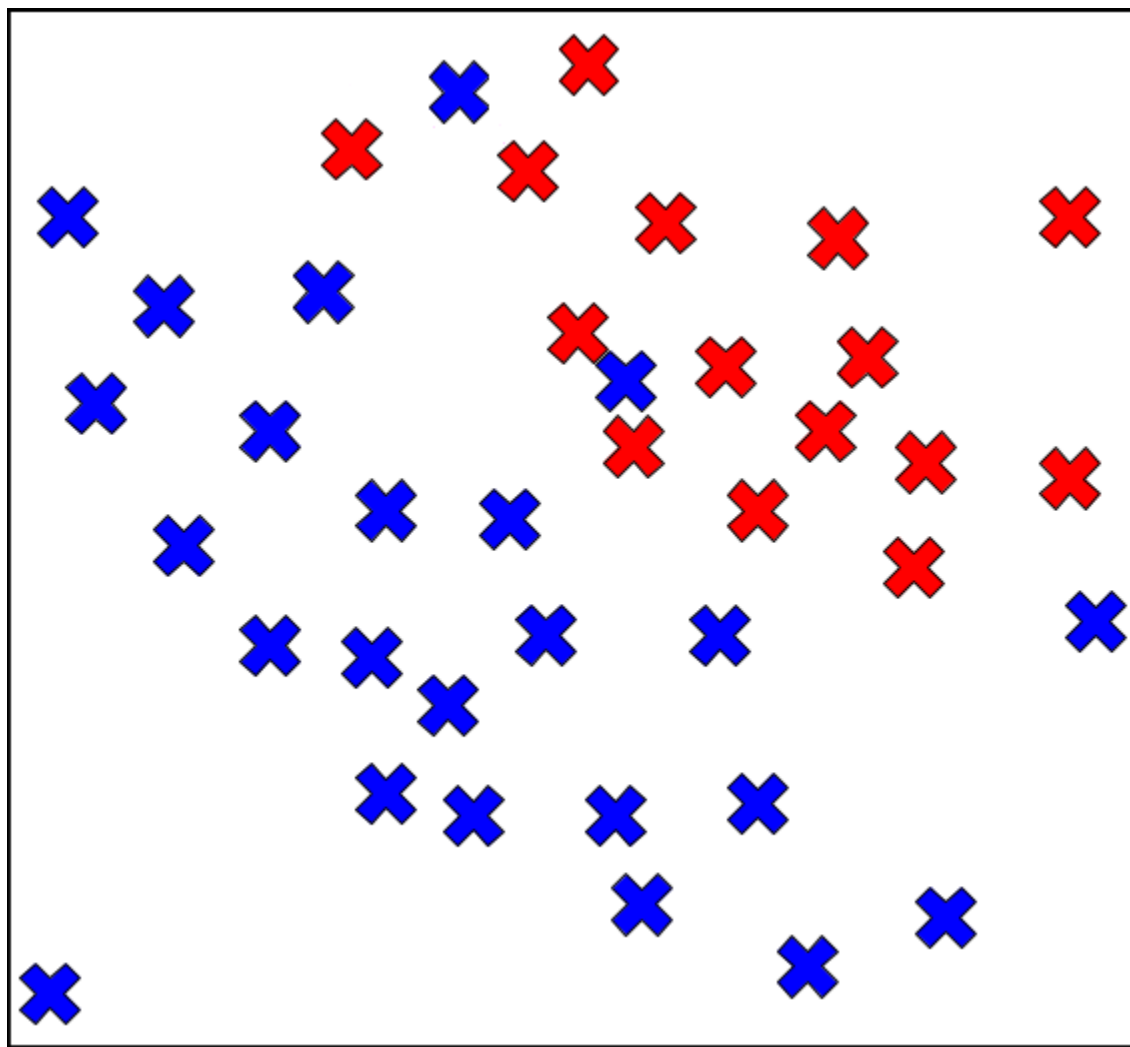
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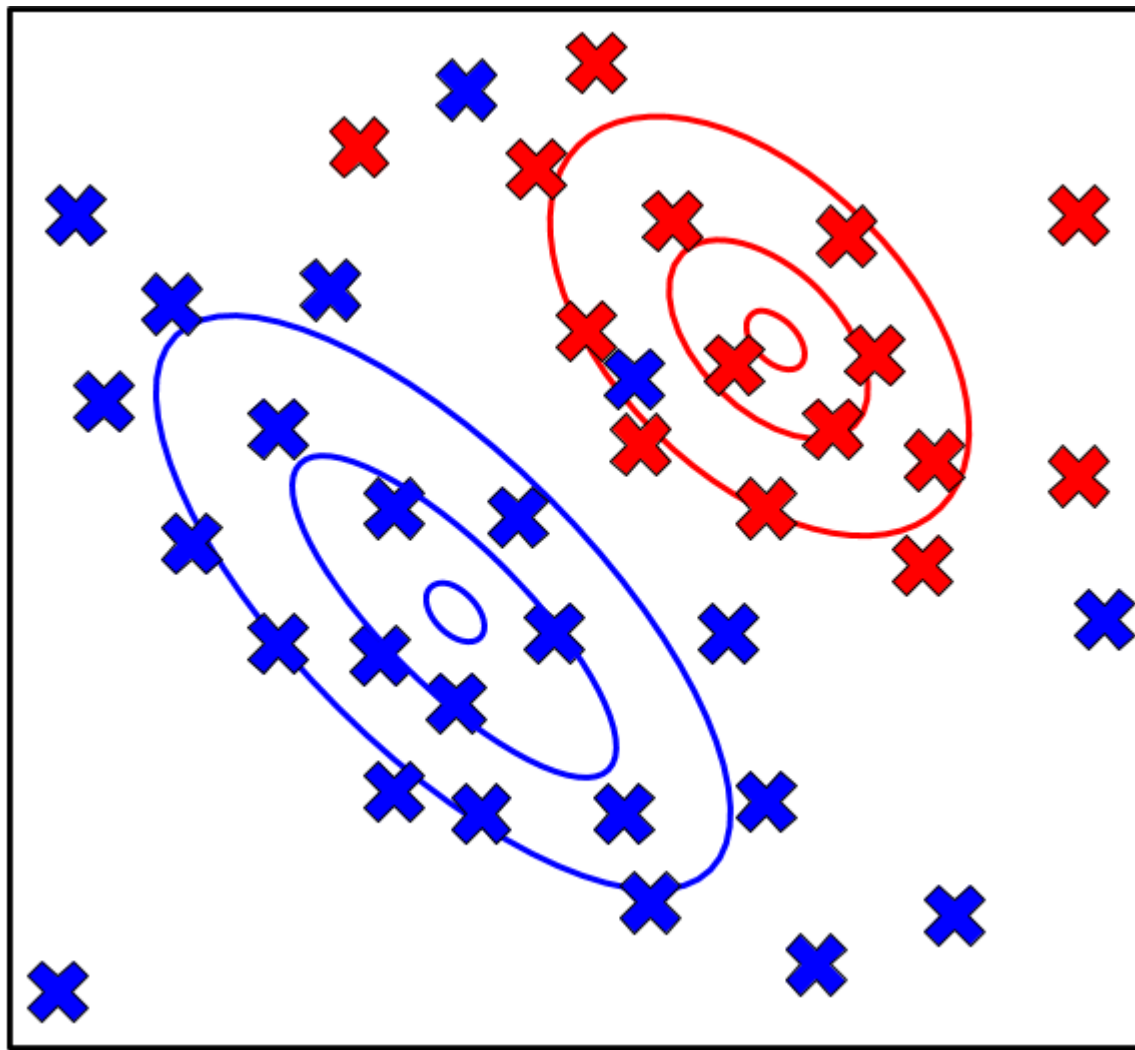
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$$p_{\theta}(x, y) = p_{\theta}(y|x) p_{\theta}(x)$$

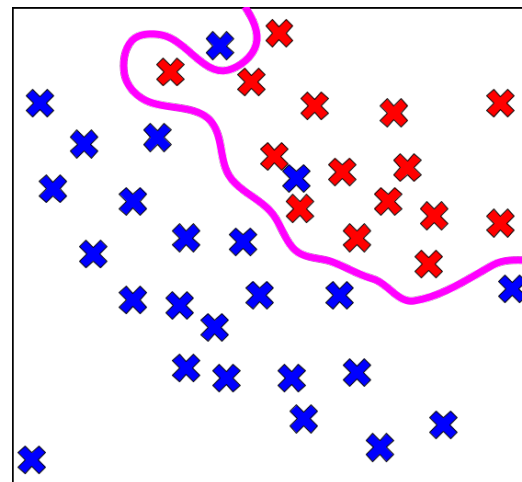
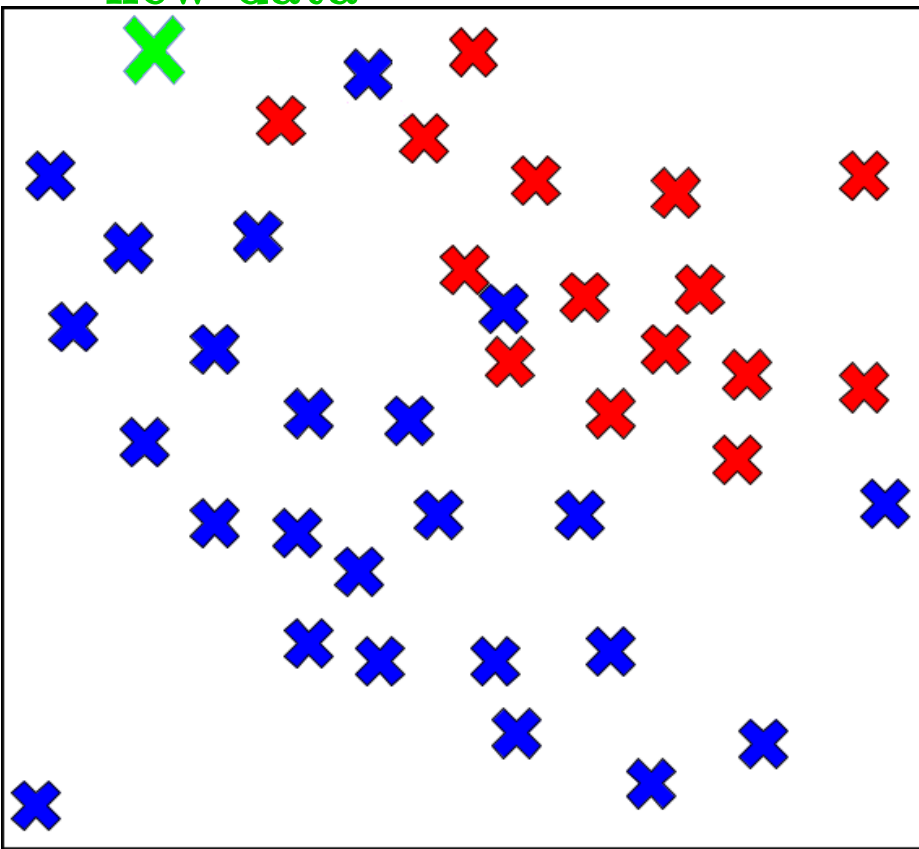
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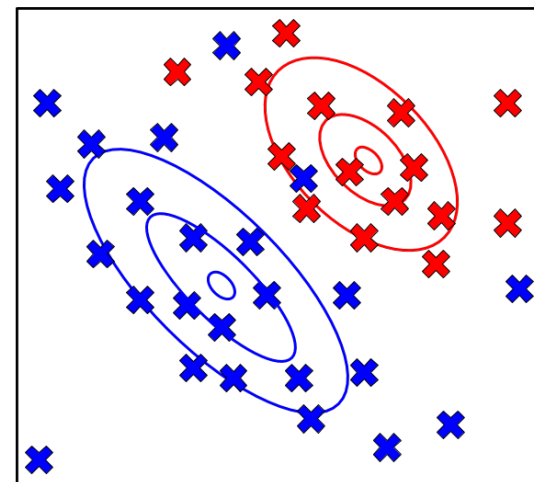
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Do we need generative modeling?

new data



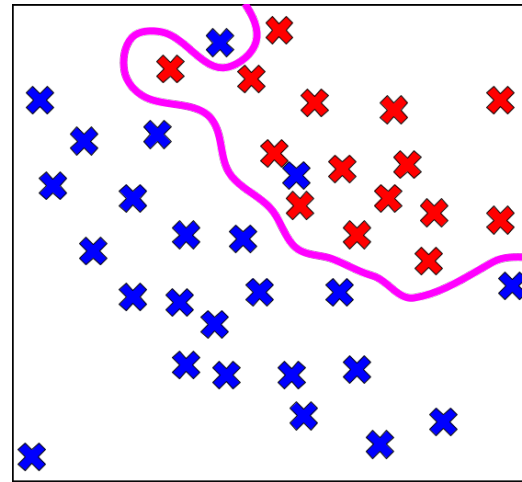
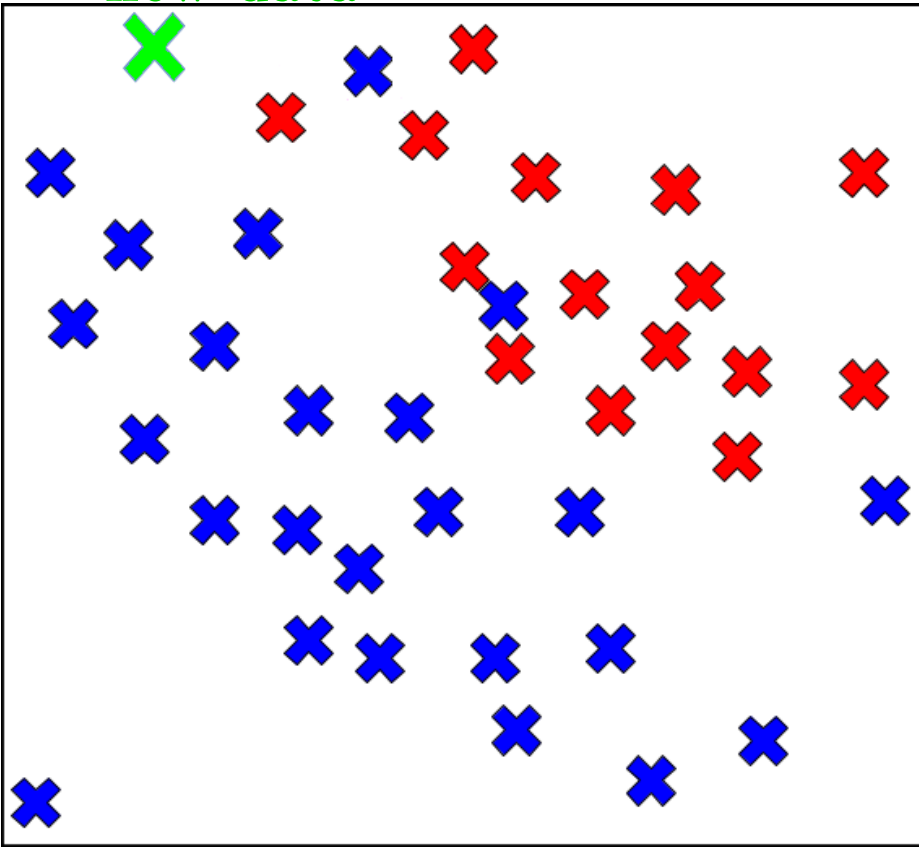
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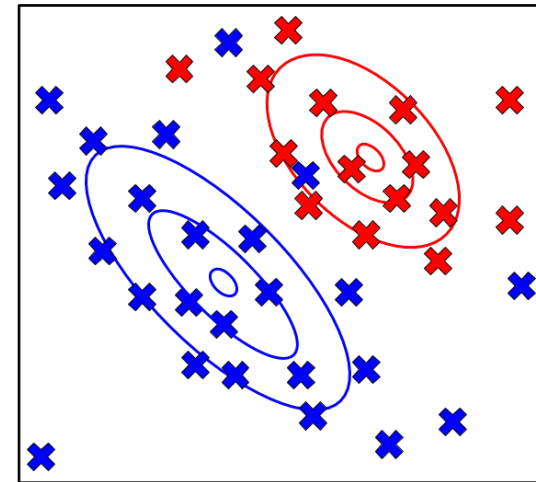
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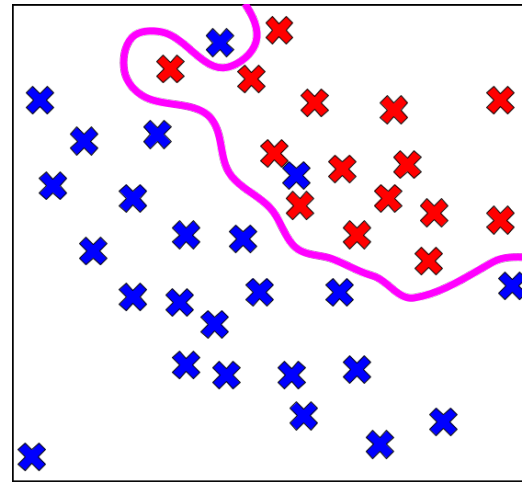
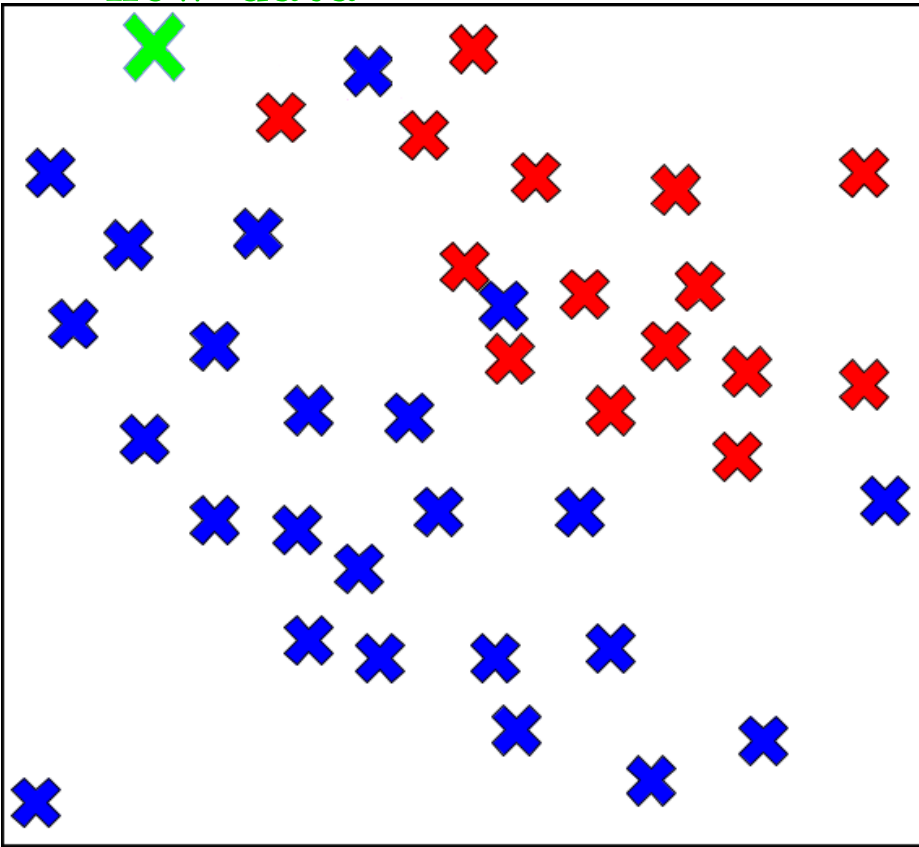
High probability
of the **blue** label.

=

Highly probable
decision!

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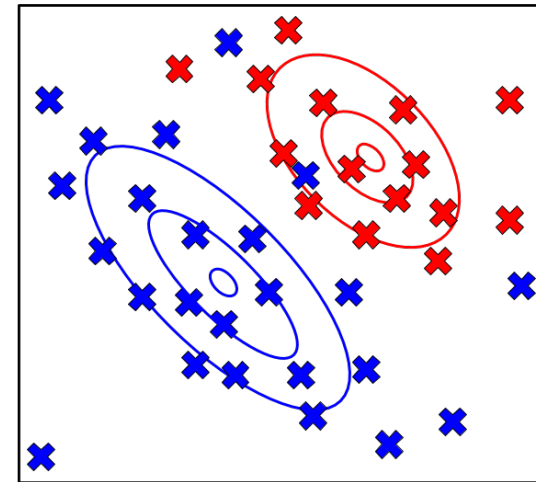


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$$p_{\theta}(x, y)$$

High probability
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x

Low probability
of the **object**.

=

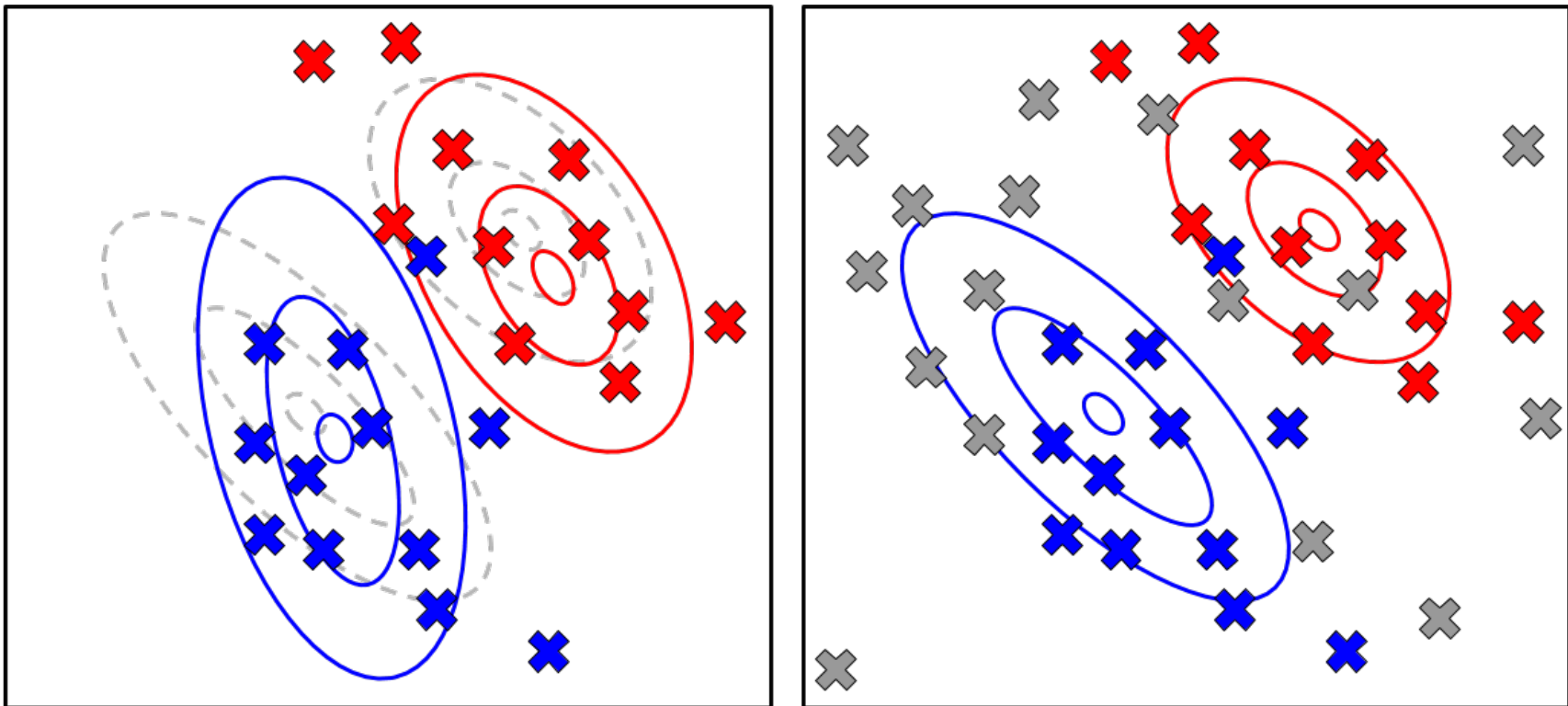
Uncertain
decision!

Generative Modeling

- Providing decision is not enough. *How to evaluate uncertainty? Distribution of y is only a part of the story.*
- Generalization problem. *Without knowing the distribution of $\mathbf{x}$ how we can generalize to new data?*
- Understanding the problem is crucial (“**What I cannot create, I do not understand**”, Richard P. Feynman). *Properly modeling data is essential to make better decisions.*

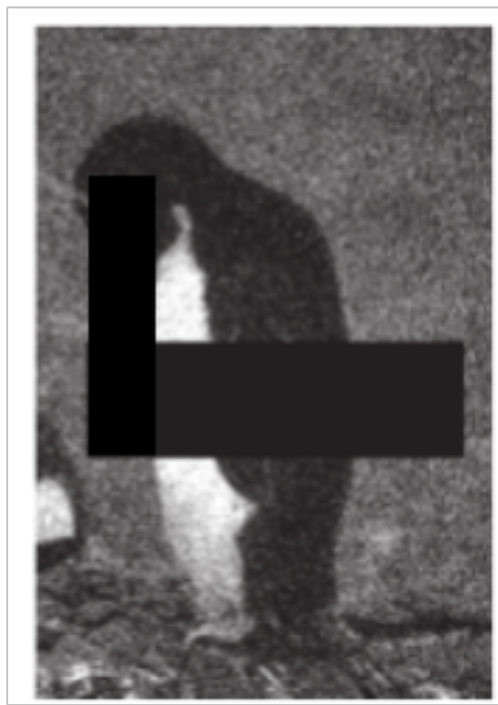
Generative Modeling

- Semi-supervised learning.
Use unlabeled data to train a better classifier.



Generative Modeling

- Handling missing or distorted data.
Reconstruct and/or denoise data.



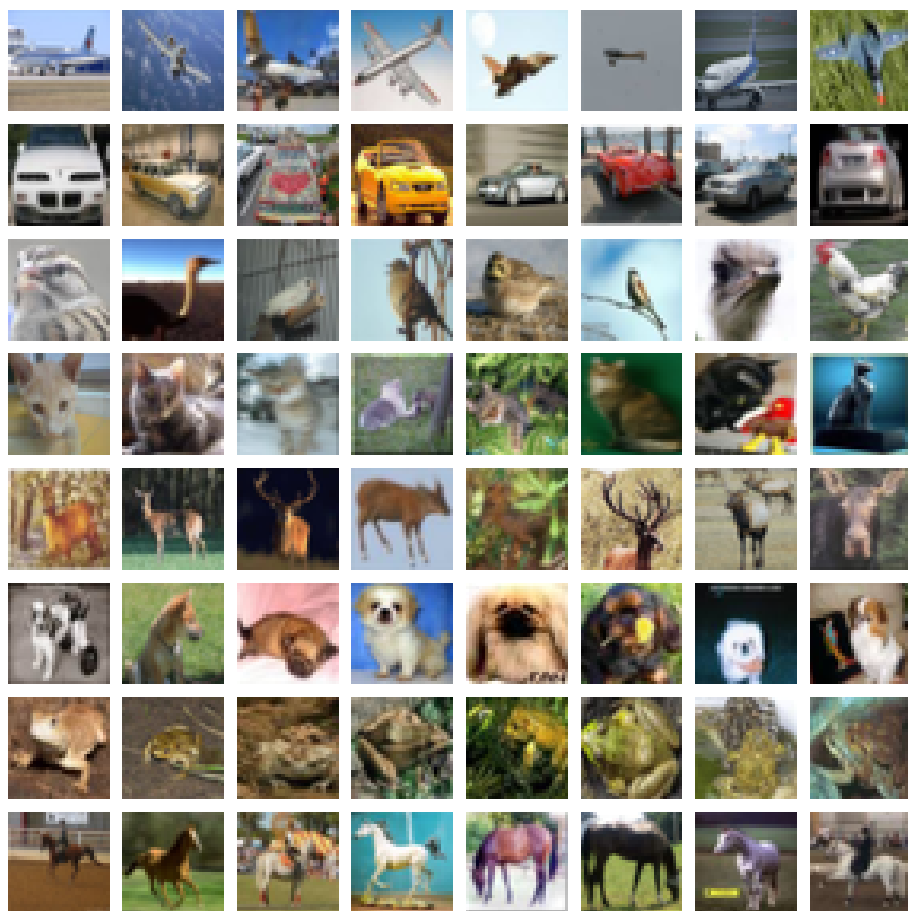
???



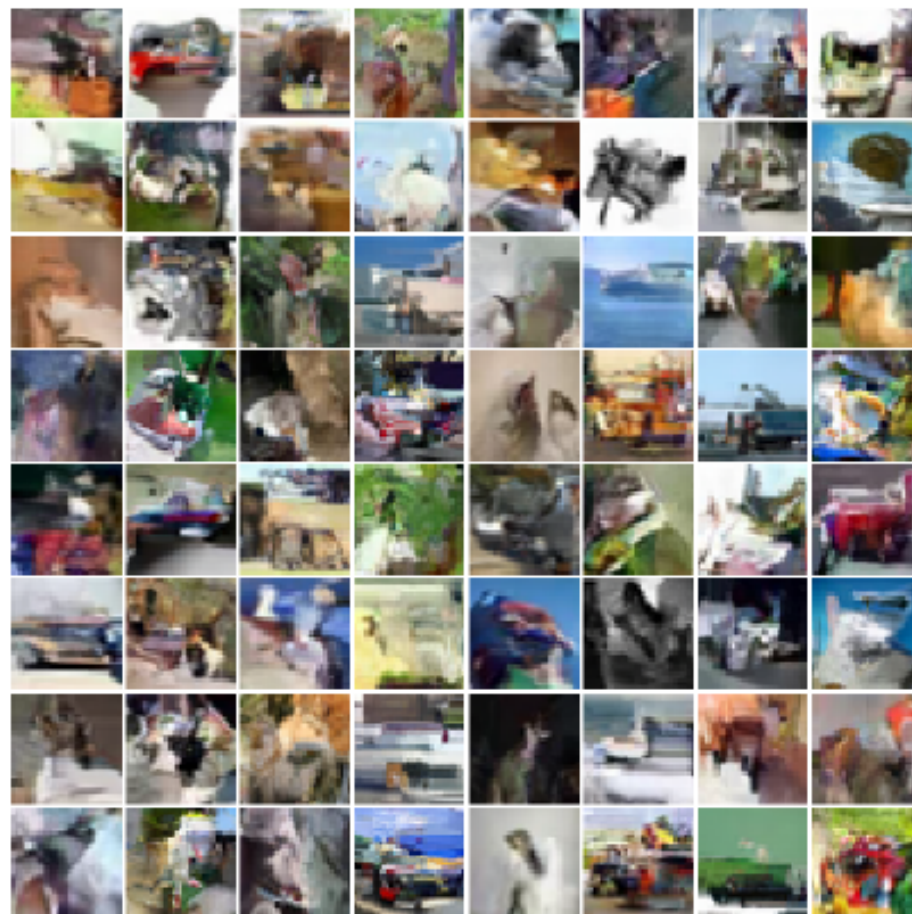
penguin!

Generative Modeling

Image generation



Real



Generated

Generative Modeling

Sequence generation

he had been unable to conceal the fact that there was a logical explanation for his inability to alter the fact that they were supposed to be on the other side of the house .

with a variety of pots strewn scattered across the vast expanse of the high ceiling , a vase of colorful flowers adorned the tops of the rose petals littered the floor and littered the floor .

atop the circular dais perched atop the gleaming marble columns began to emerge from atop the stone dais, perched atop the dais .

Generated

How to formulate a generative model?

Modeling in high-dimensional space is difficult.



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→ modeling all dependencies among pixels.

$$p(x) = \prod_{d=1}^c \psi_c(x_c)$$

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very inefficient!

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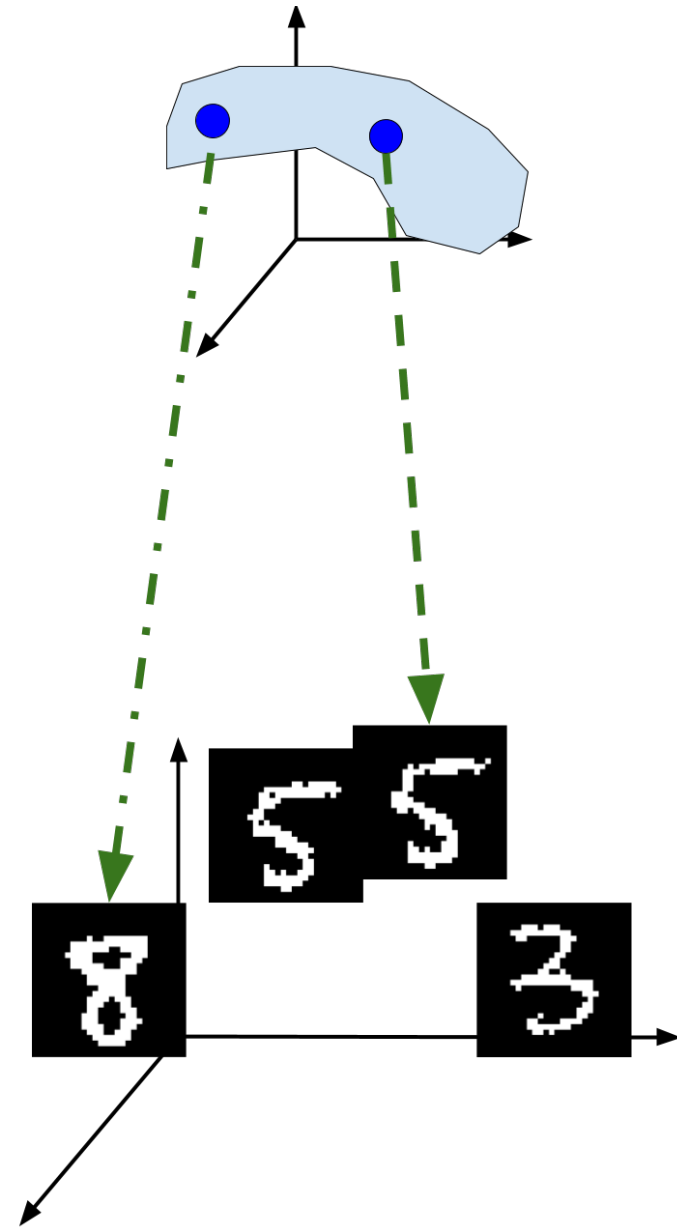
very inefficient!

A possible **solution**? → **Latent variable models**

Latent Variable Models

- Latent variable model:

$$p(\mathbf{x}) = \int p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\lambda}(\mathbf{z}) d\mathbf{z}$$



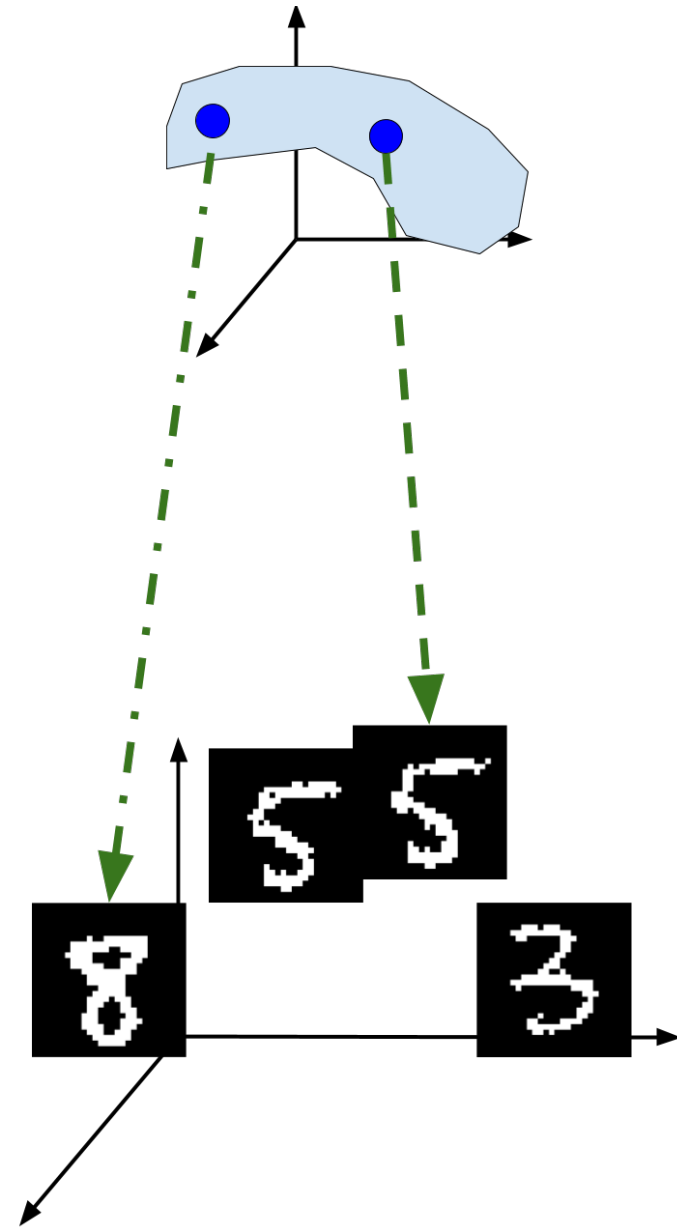
Latent Variable Models

- Latent variable model:

$$p(\mathbf{x}) = \int p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\lambda}(\mathbf{z}) d\mathbf{z}$$

First sample $\mathbf{z}$.

Second, sample $\mathbf{x}$ for given $\mathbf{z}$.



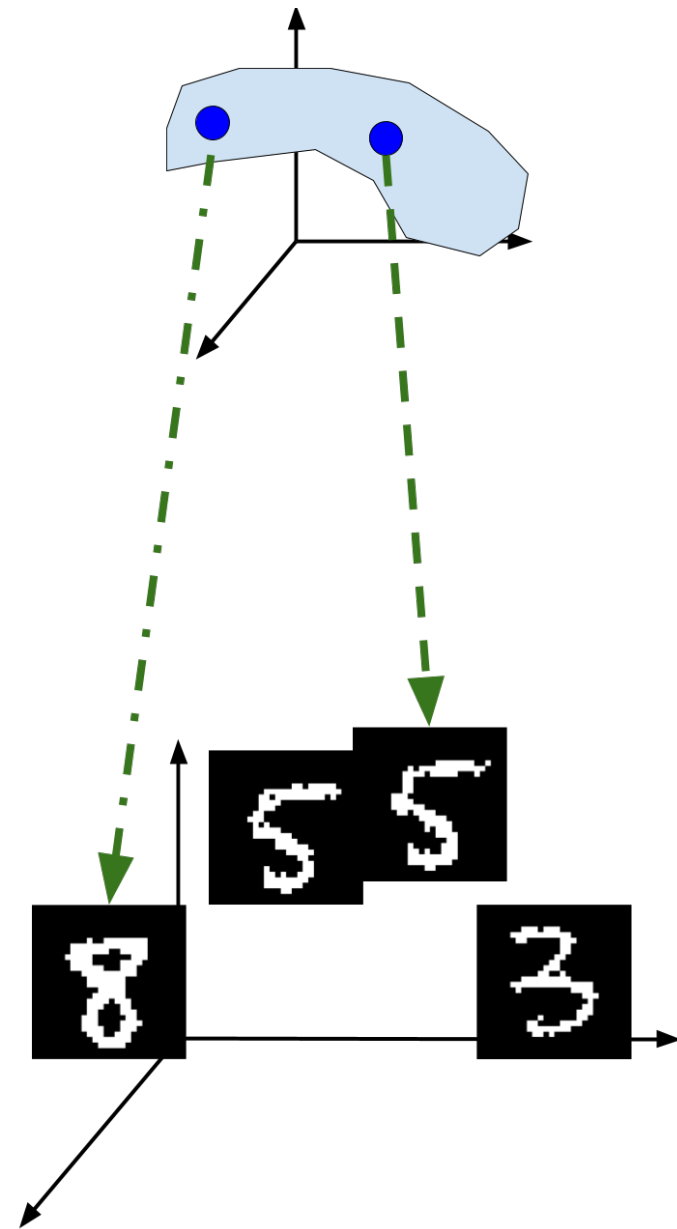
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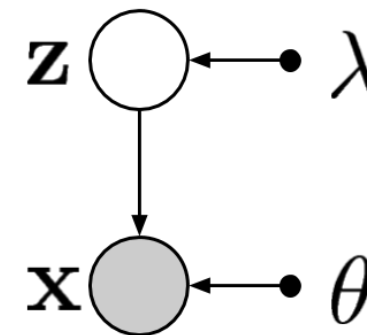
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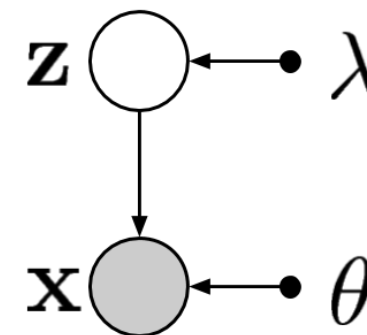


- If $p_{\theta}(\mathbf{x}|\mathbf{z}) = \mathcal{N}(\mathbf{W}\mathbf{z} + \mathbf{b}, \Psi)$ and $p_{\lambda}(\mathbf{z}) = \mathcal{N}(\mu_0, \Sigma_0)$,
then $\rightarrow$ **Factor Analysis**.
- What if we take a non-linear transformation of $\mathbf{z}$?
 $\rightarrow$ *an infinite mixture of Gaussians*.

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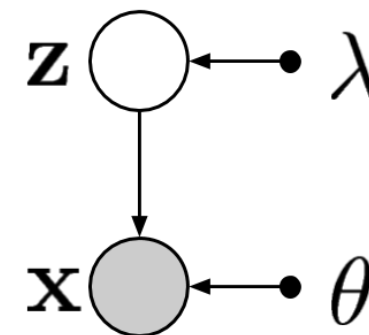
Convenient but limiting!

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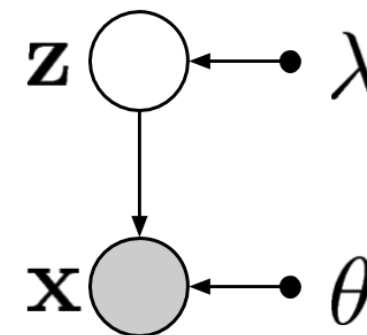


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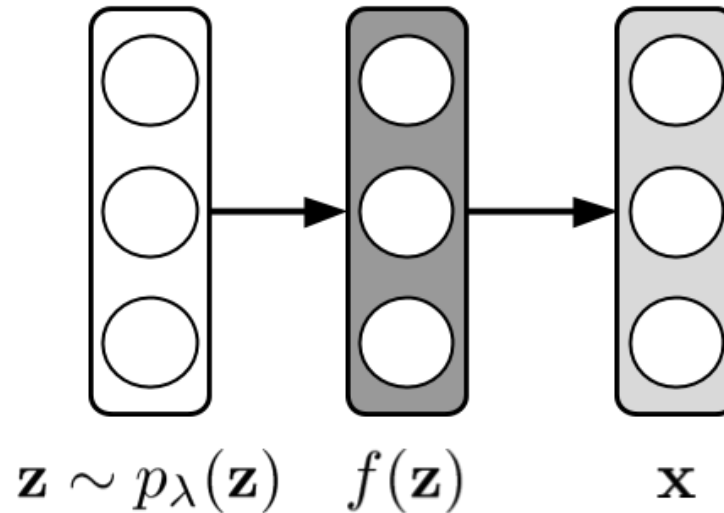
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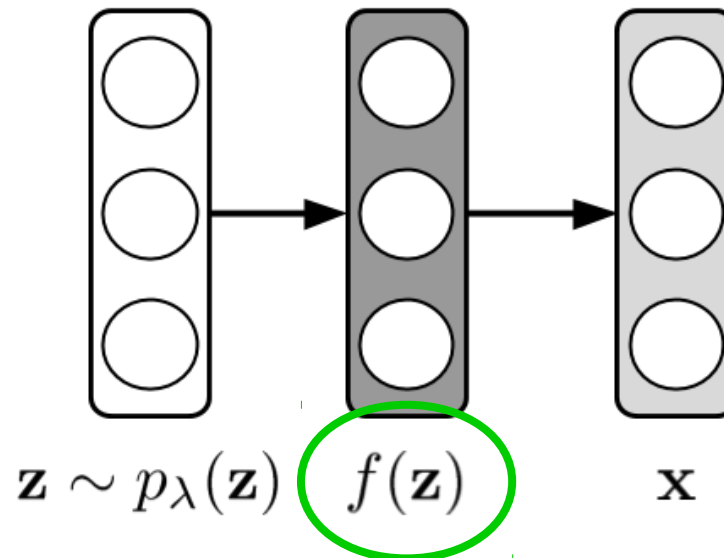


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Neural network

Deep Generative Models (DGM): Density Network

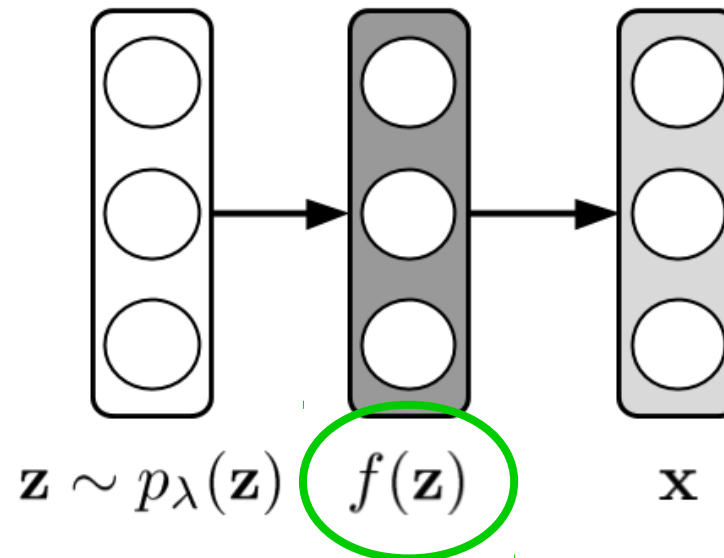


DGM: Density Network



Neural Network

DGM: Density Network



Neural Network

How to train this model?!

DGM: Density Network

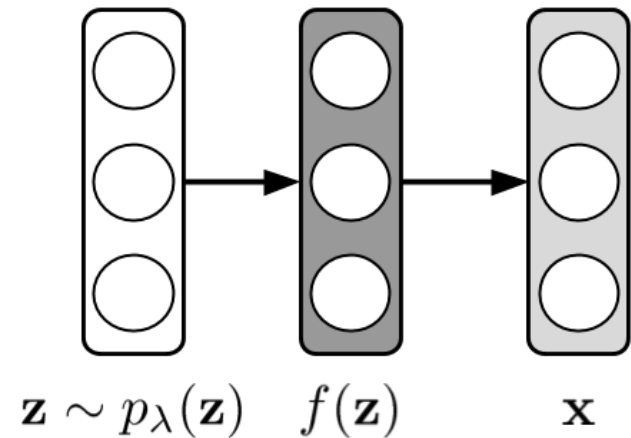
- MC approximation:

$$\log p(\mathbf{x}) = \log \int p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\lambda}(\mathbf{z}) d\mathbf{z}$$

$$\approx \log \frac{1}{S} \sum_{s=1}^S \exp \left(\log p_{\theta}(\mathbf{x}|\mathbf{z}_s) \right)$$

where:

$$\mathbf{z}_s \sim p_{\lambda}(\mathbf{z})$$



DGM: Density Network

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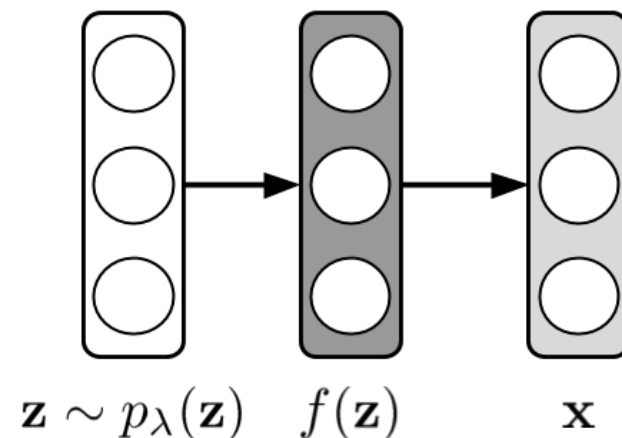
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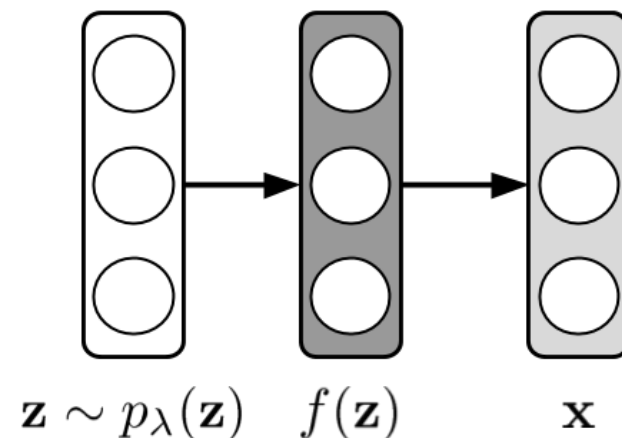
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**It scales badly in high
dimensional cases!**



DGM: Density Network

PROS

Log-likelihood approach

Easy sampling

Training using gradient-based methods

CONS

Requires explicit models

Fails in high dim. cases

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Can we do better?

DGM: Generative Adversarial Nets

Let image two agents:

DGM: Generative Adversarial Nets

Let image two agents:



A fraud

DGM: Generative Adversarial Nets

Let image two agents:



A fraud



An art expert

DGM: Generative Adversarial Nets

Let image two agents:



A fraud



... and a real artist



An art expert

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The fraud aims
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DGM: Generative Adversarial Nets

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The fraud aims to copy the real artist and cheat the art expert.

The expert assesses a painting and gives her opinion.



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DGM: Generative Adversarial Nets

Let image two agents:



A fraud



... and a real artist

The fraud aims to copy the real artist and cheat the art expert.

The expert assesses a painting and gives her opinion.

The fraud learns and tries to fool the expert.



An art expert

DGM: Generative Adversarial Nets

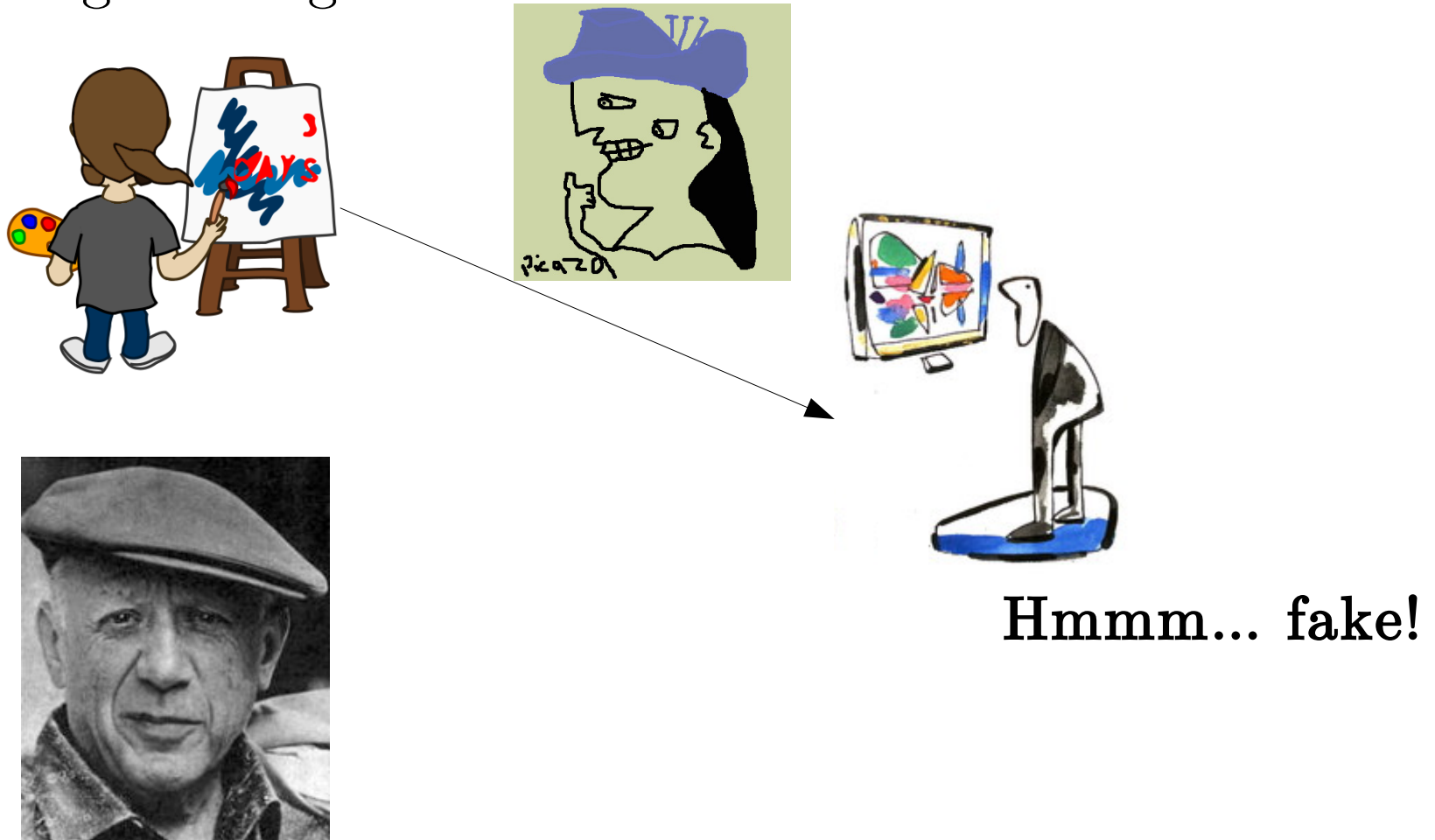
Let image two agents:



Hmmm... fake!

DGM: Generative Adversarial Nets

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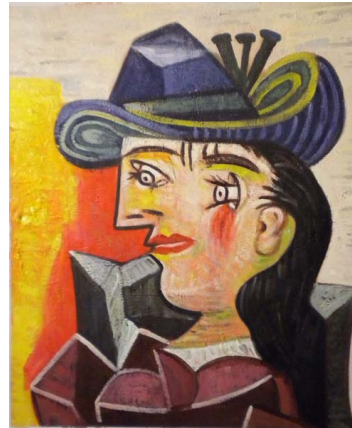
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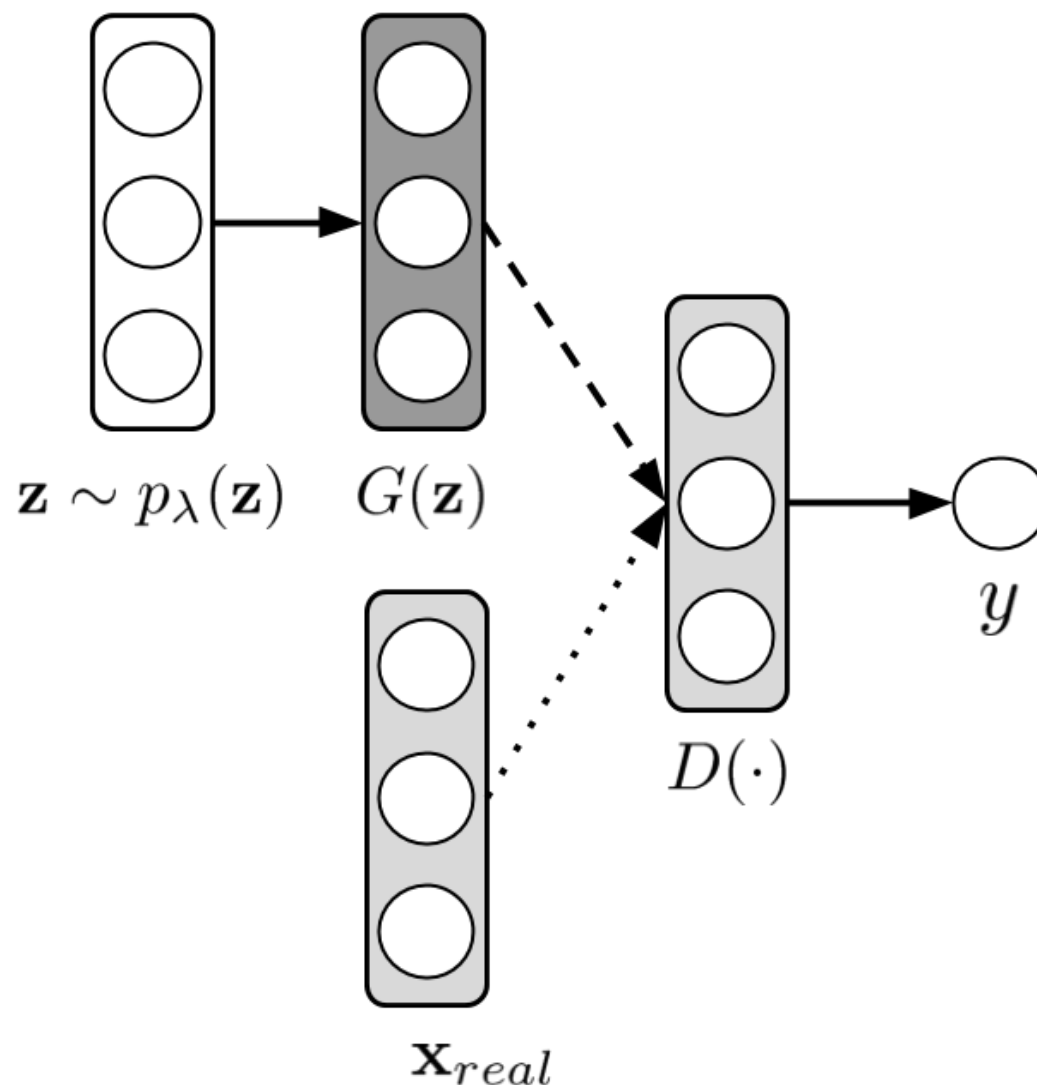
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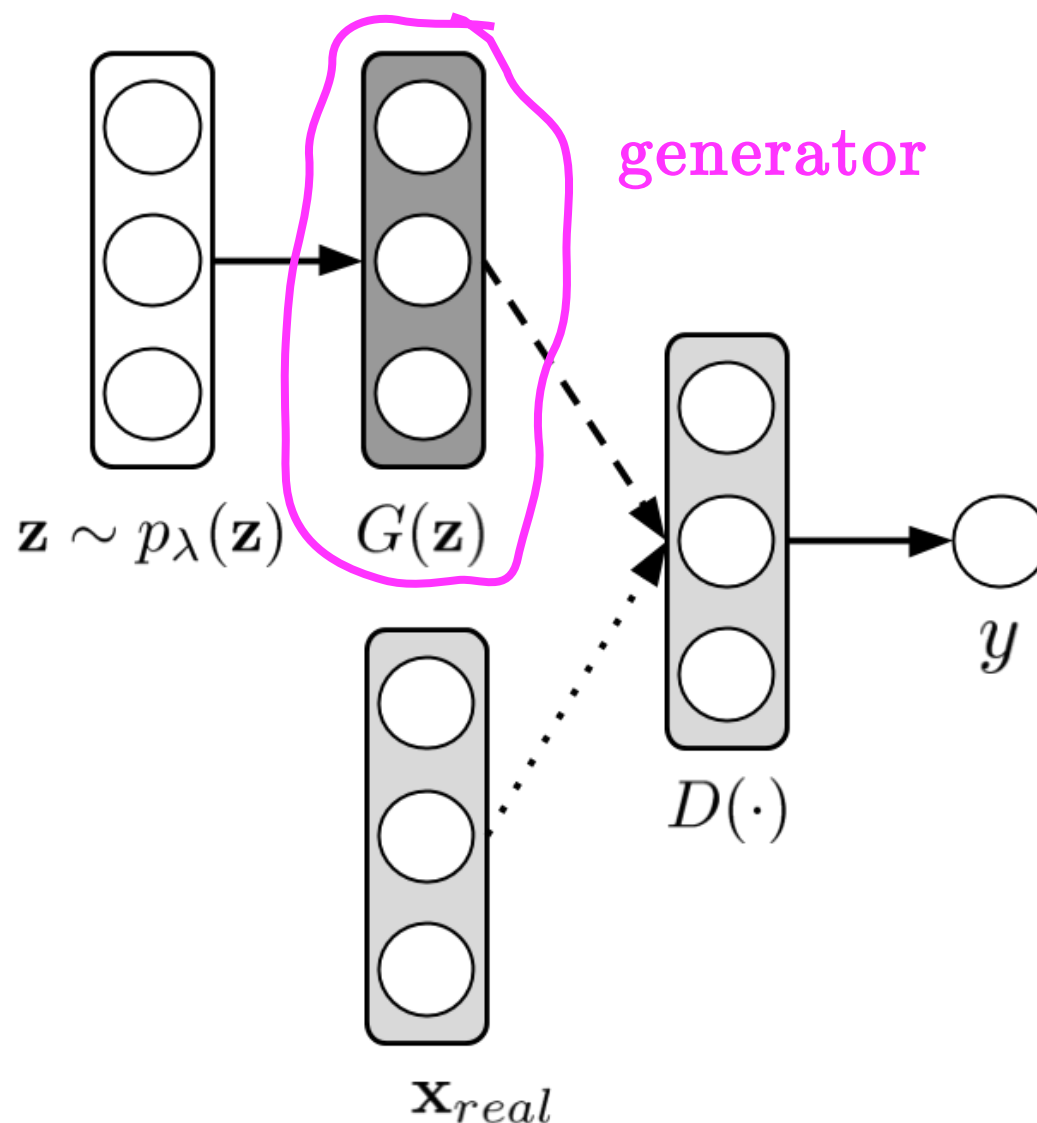


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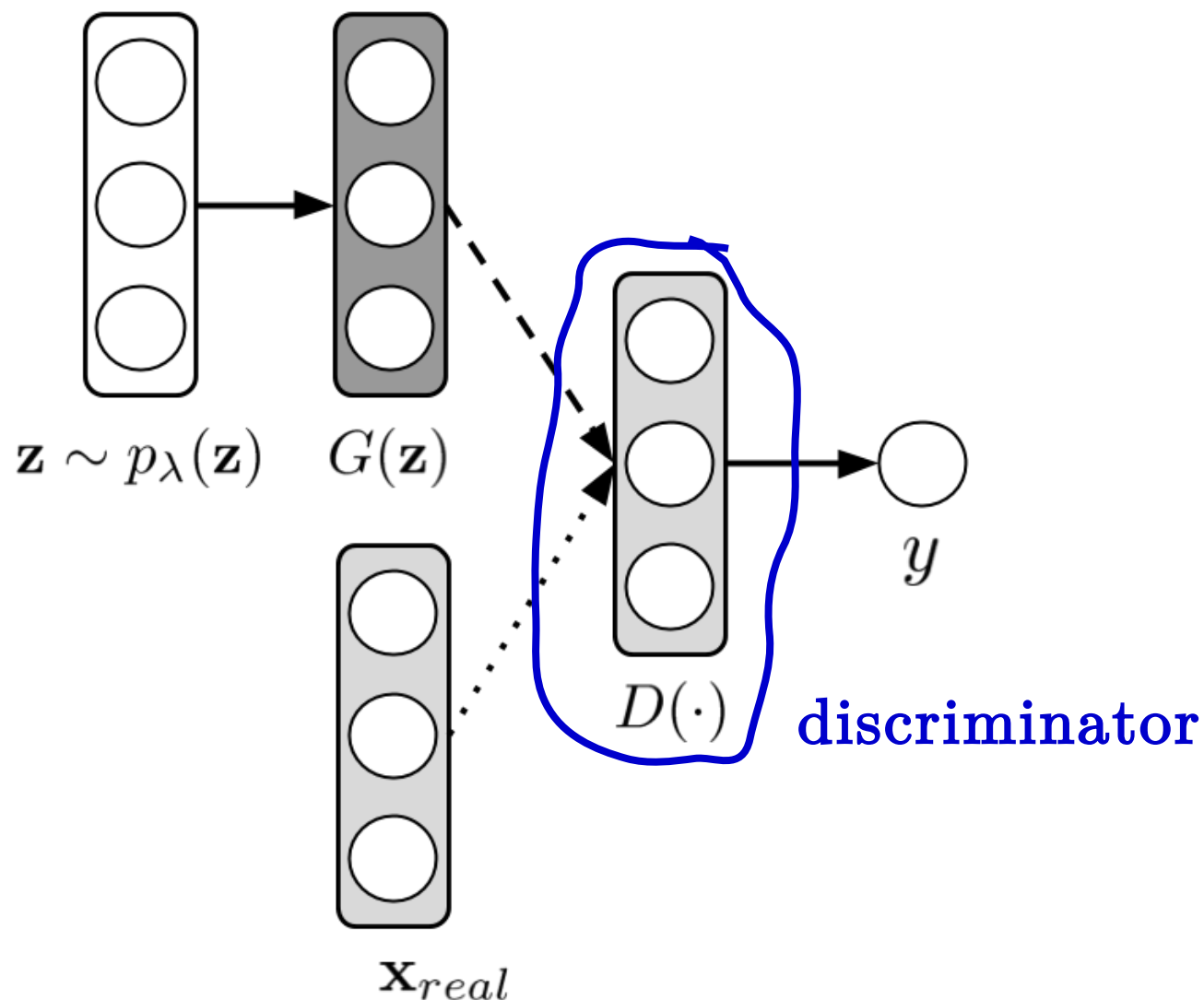
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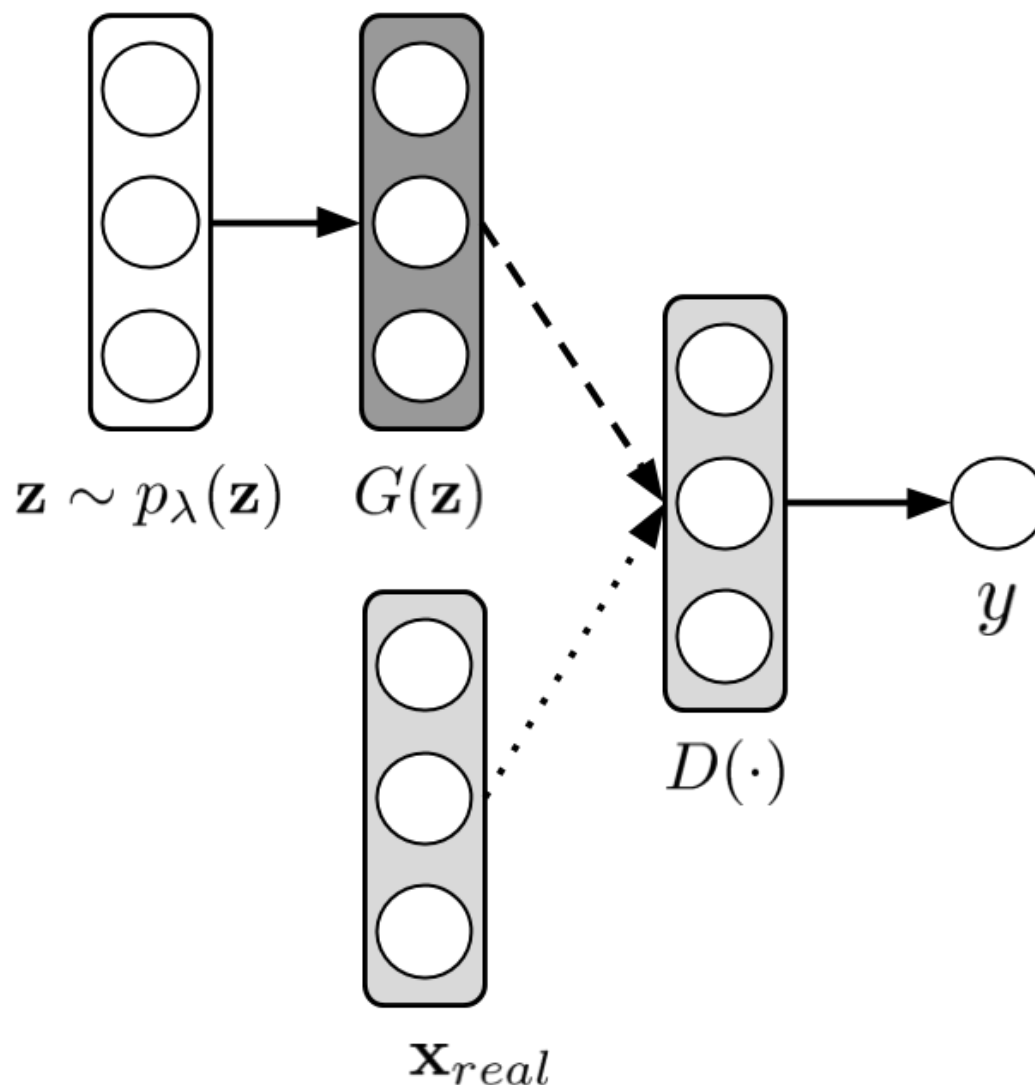


DGM: Generative Adversarial Nets



DGM: Generative Adversarial Nets

1. Sample $\mathbf{z}$.
2. Generate $G(\mathbf{z})$.
3. Discriminate whether given image is **real** or **fake**.



DGM: Generative Adversarial Nets

Formally, the problem is the following:

$$\min_G \max_D \mathbb{E}_{\mathbf{x} \sim p_{real}} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

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Minimize wrt. generator

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Maximize wrt. discriminator

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Once we converge, we can generate images that are almost **indistinguishable** from real images.

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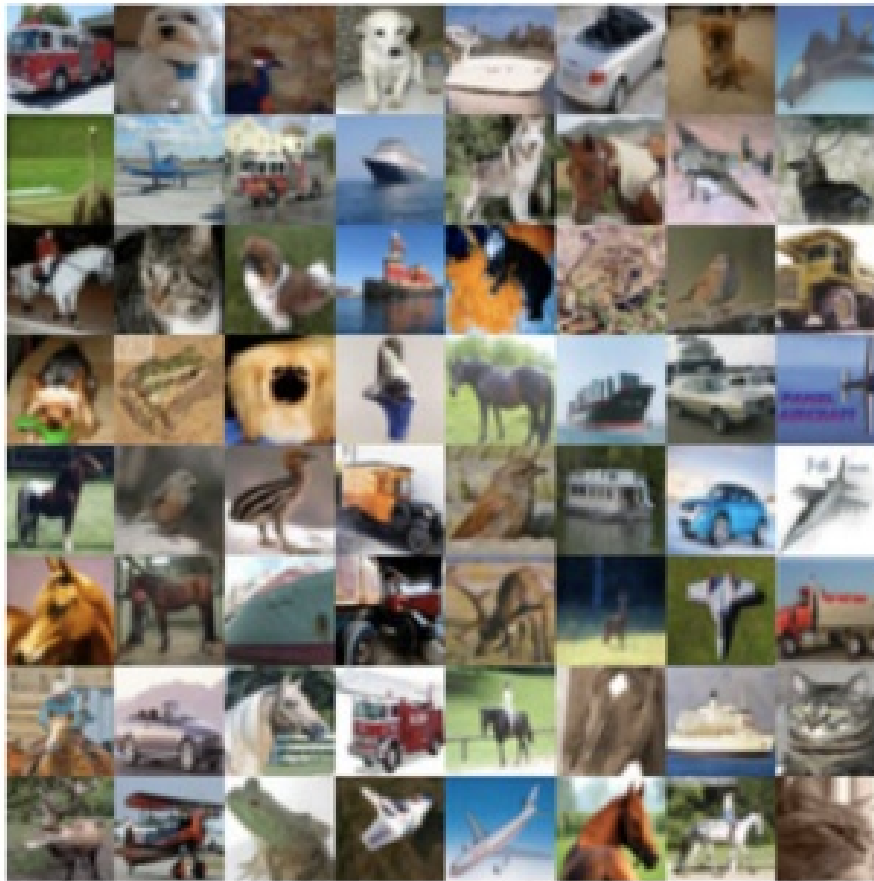
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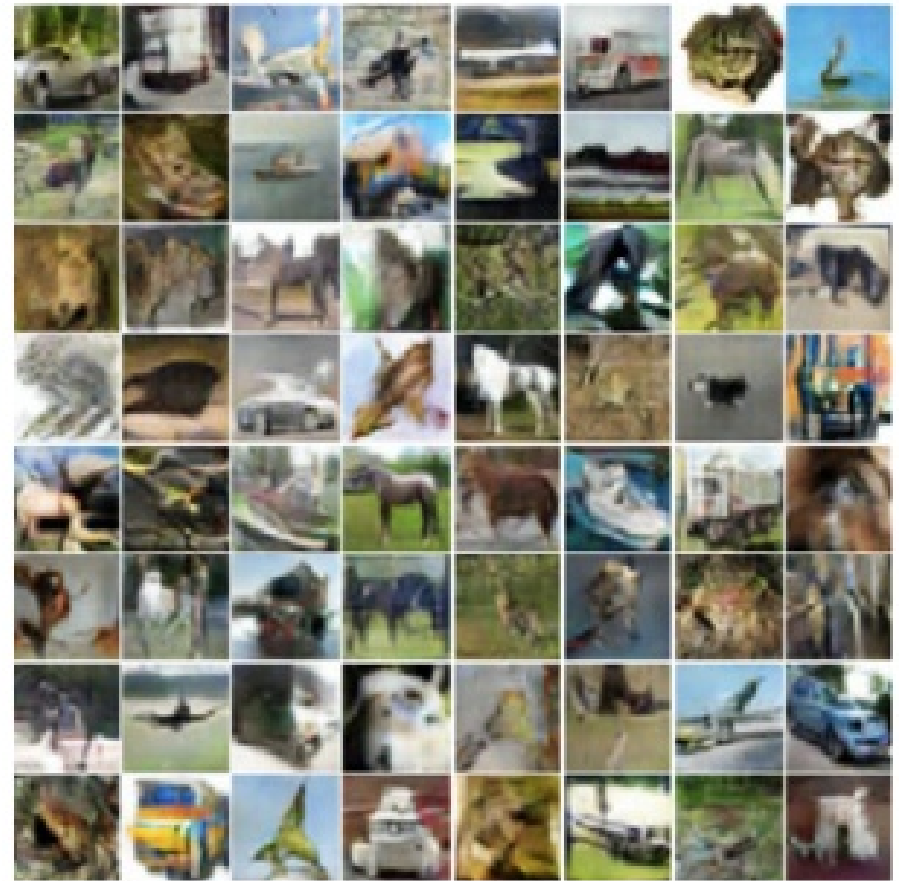
Once we converge, we can generate images that are almost **indistinguishable** from real images.

BUT training is very unstable...

DGM: Generative Adversarial Nets



Training Data



Samples

DGM: GANs

PROS

Allows **implicit** models

Easy **sampling**

Training using **gradient-based methods**

Works in **high dim. cases**

CONS

Unstable training

Does not correspond to likelihood solution

No clear way for **quantitative assessment**

Missing mode problem

DGM: Wasserstein GAN

We can consider an earth-mover distance to formulate GAN-like optimization problem as follows:

$$\min_G \max_{D \in \mathcal{W}} \mathbb{E}_{\mathbf{x} \sim p_{real}} [D(\mathbf{x})] - \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} [D(G(\mathbf{z}))]$$

where the discriminator is a 1-Lipshitz function.

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It means we need to **clip** weights of the discriminator,
i.e.,

`clip(weights, -c, c).`

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where the discriminator is a 1-Lipshitz function.

Wasserstein GAN **stabilizes training** (but other problems **remain**).

DGM: More GANs (selected)

Deep convolutional generative adversarial networks

Radford, A., Metz, L., & Chintala, S. (2015). Unsupervised representation learning with deep convolutional generative adversarial networks. arXiv preprint arXiv:1511.06434.

Auxiliary classifier GANs

Odena, A., Olah, C., & Shlens, J. (2016). Conditional image synthesis with auxiliary classifier gans. arXiv preprint arXiv:1610.09585.

From optimal transport to generative modeling: the VEGAN cookbook

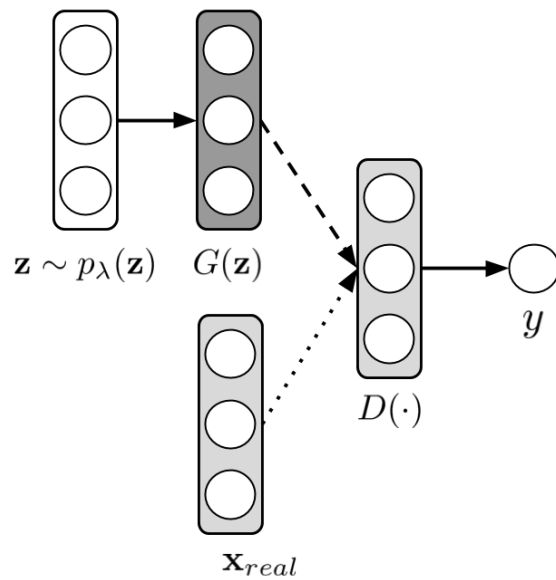
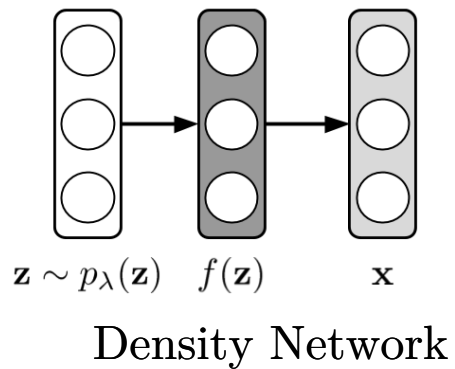
Bousquet, O., Gelly, S., Tolstikhin, I., Simon-Gabriel, C. J., & Schoelkopf, B. (2017). From optimal transport to generative modeling: the VEGAN cookbook. arXiv preprint arXiv:1705.07642.

Bidirectional Generative Adversarial Networks

Donahue, J., Krähenbühl, P., & Darrell, T. (2016). Adversarial feature learning. arXiv preprint arXiv:1605.09782.

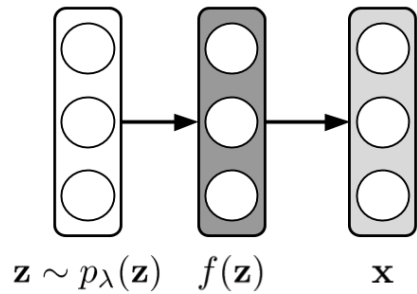
Questions?

DGM: so far we have



Generative Adversarial Net

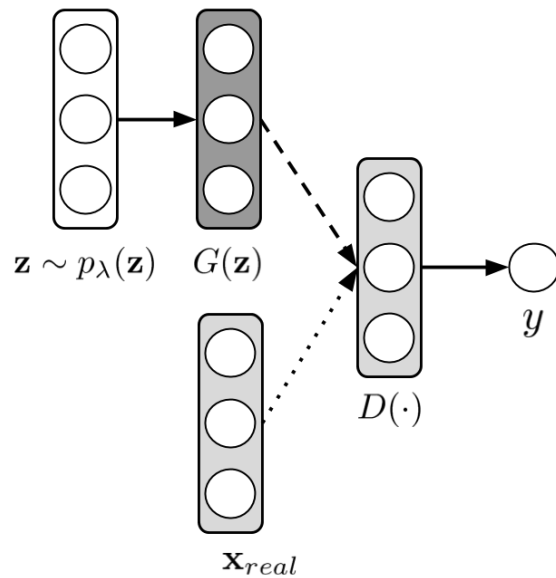
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Density Network

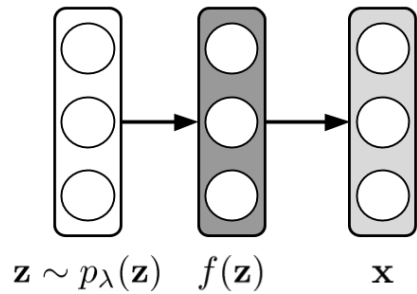
Works only for low dim. cases...

Inefficient training...



Generative Adversarial Net

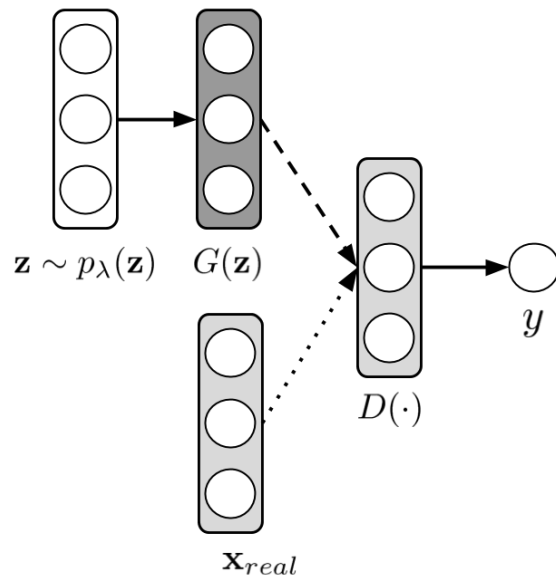
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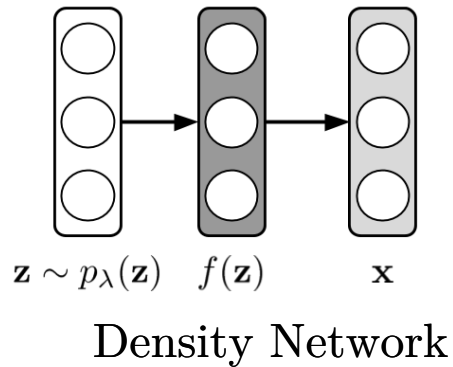
Inefficient training...



Generative Adversarial Net

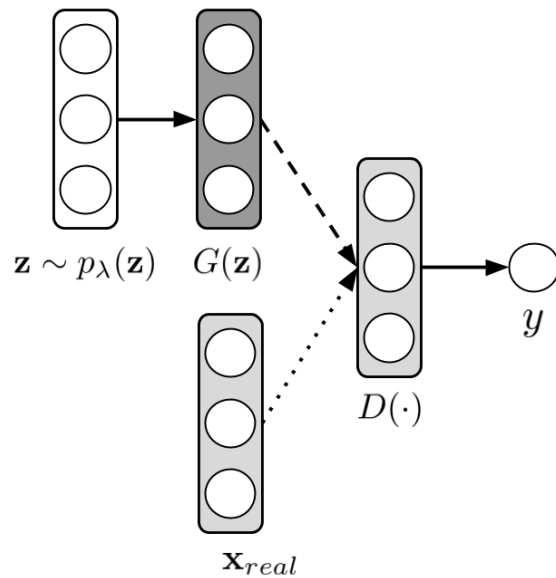
Works for high dim. cases!

DGM: so far we have



Works only for low dim. cases...

Inefficient training...



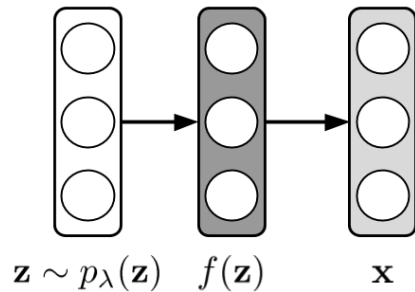
Generative Adversarial Net

Works for high dim. cases!

Doesn't train a distribution...

Unstable training...

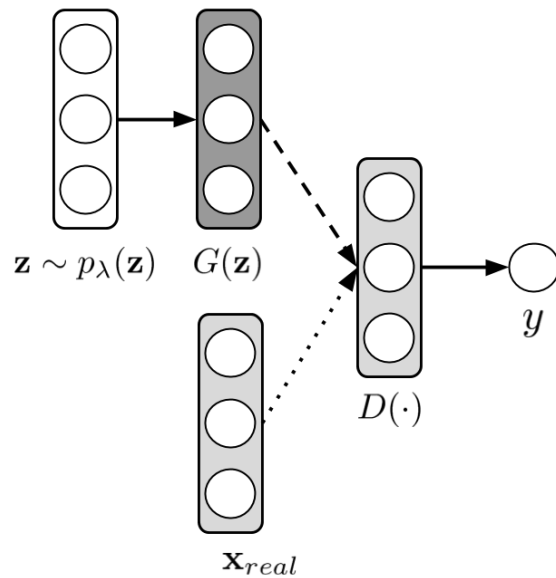
DGM: so far we have



Density Network

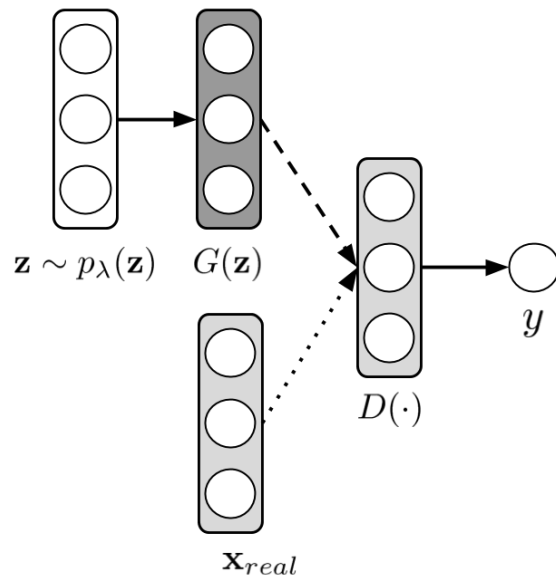
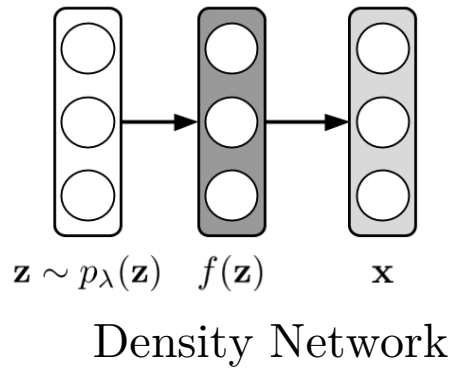
QUESTION

Can we stick to the log-likelihood approach but with a simple training procedure?

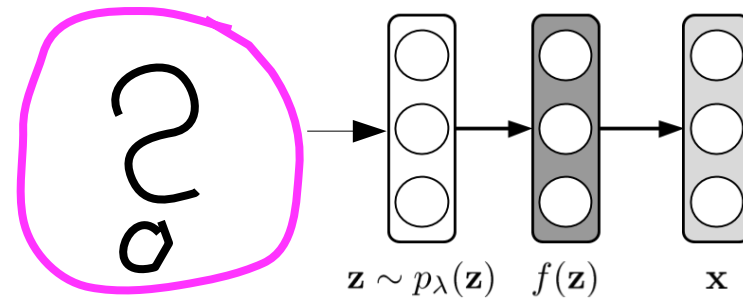


Generative Adversarial Net

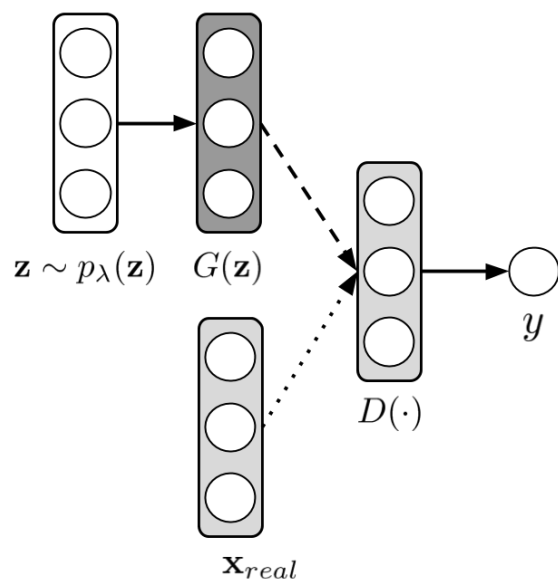
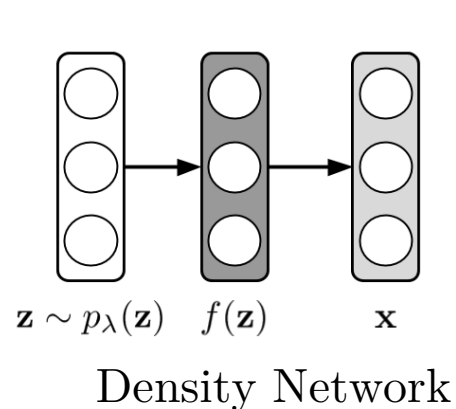
DGM: so far we have



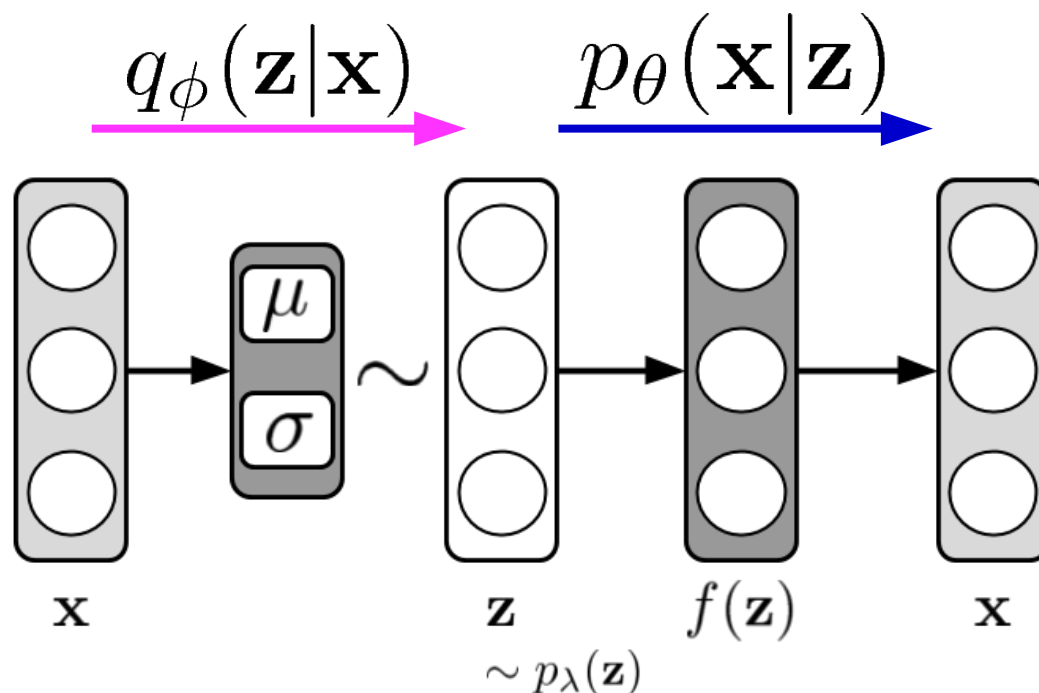
Generative Adversarial Net



DGM: Variational Auto-Encoder

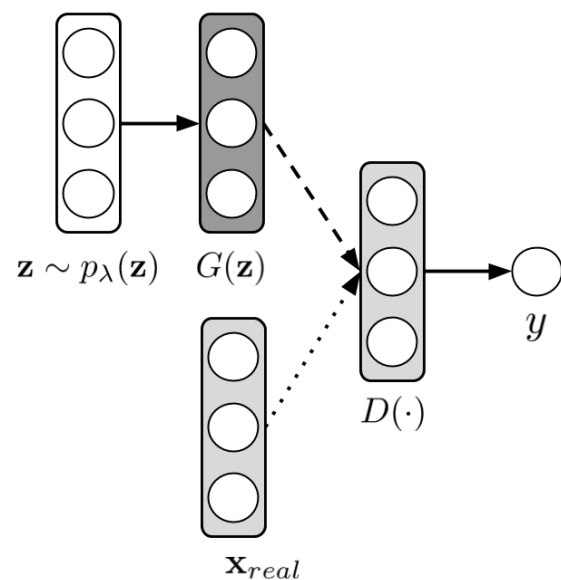
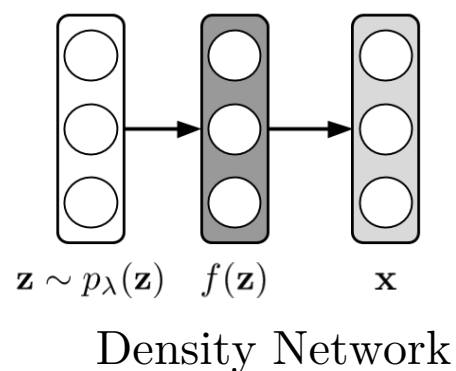


Generative Adversarial Net

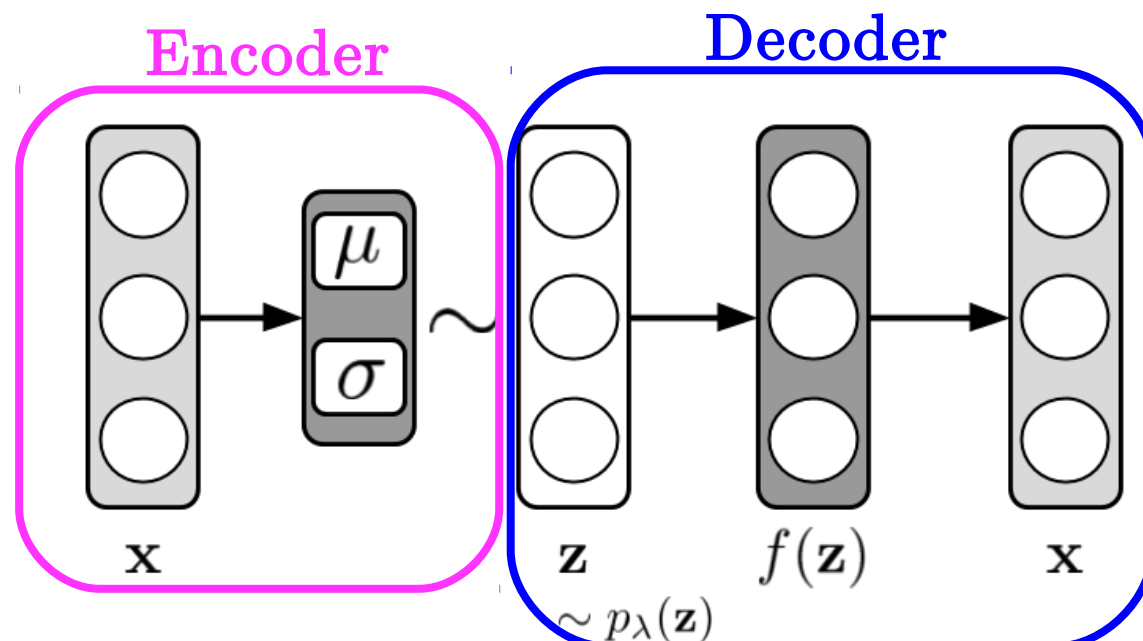


Variational Auto-Encoder

DGM: Variational Auto-Encoder

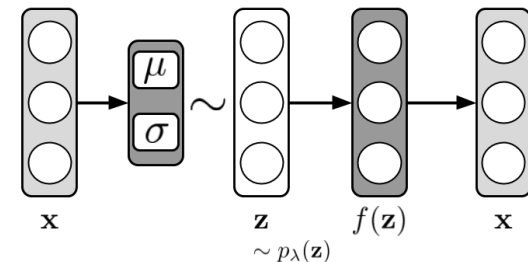


Generative Adversarial Net



Variational Auto-Encoder

DGM: Variational Auto-Encoder



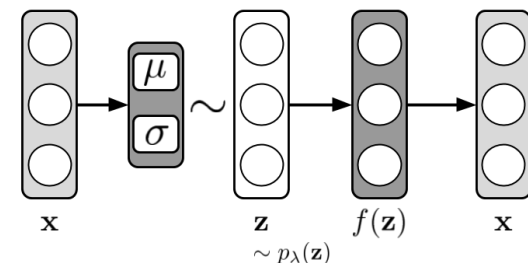
$$\log p(\mathbf{x}) = \log \int p_\theta(\mathbf{x}|\mathbf{z}) p_\lambda(\mathbf{z}) d\mathbf{z}$$

$$= \log \int \frac{q_\phi(\mathbf{z}|\mathbf{x})}{q_\phi(\mathbf{z}|\mathbf{x})} p_\theta(\mathbf{x}|\mathbf{z}) p_\lambda(\mathbf{z}) d\mathbf{z}$$

$$\geq \int q_\phi(\mathbf{z}|\mathbf{x}) \log \frac{p_\theta(\mathbf{x}|\mathbf{z}) p_\lambda(\mathbf{z})}{q_\phi(\mathbf{z}|\mathbf{x})} d\mathbf{z}$$

$$= \mathbb{E}_{\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] - \text{KL}[q_\phi(\mathbf{z}|\mathbf{x}) || p_\lambda(\mathbf{z})]$$

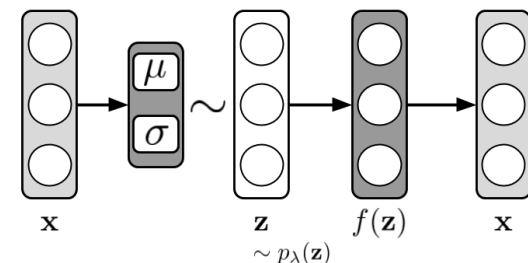
DGM: Variational Auto-Encoder



$$\begin{aligned}\log p(\mathbf{x}) &= \log \int p_\theta(\mathbf{x}|\mathbf{z}) p_\lambda(\mathbf{z}) d\mathbf{z} \\ &= \log \int \frac{q_\phi(\mathbf{z}|\mathbf{x})}{q_\phi(\mathbf{z}|\mathbf{x})} p_\theta(\mathbf{x}|\mathbf{z}) p_\lambda(\mathbf{z}) d\mathbf{z} \\ &\geq \int q_\phi(\mathbf{z}|\mathbf{x}) \log \frac{p_\theta(\mathbf{x}|\mathbf{z}) p_\lambda(\mathbf{z})}{q_\phi(\mathbf{z}|\mathbf{x})} d\mathbf{z} \\ &= \mathbb{E}_{\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] - \text{KL}[q_\phi(\mathbf{z}|\mathbf{x}) || p_\lambda(\mathbf{z})]\end{aligned}$$

Variational posterior

DGM: Variational Auto-Encoder



$$\log p(\mathbf{x}) = \log \int p_\theta(\mathbf{x}|\mathbf{z}) p_\lambda(\mathbf{z}) d\mathbf{z}$$

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$$\geq \int q_\phi(\mathbf{z}|\mathbf{x}) \log \frac{p_\theta(\mathbf{x}|\mathbf{z}) p_\lambda(\mathbf{z})}{q_\phi(\mathbf{z}|\mathbf{x})} d\mathbf{z}$$

$$= \underbrace{\mathbb{E}_{\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction error}} - \underbrace{\text{KL}[q_\phi(\mathbf{z}|\mathbf{x}) || p_\lambda(\mathbf{z})]}_{\text{Regularization}}$$

DGM: Variational Auto-Encoder

Our objective is **the evidence lower bound**.

$$\log p(\mathbf{x}) \geq \mathbb{E}_{\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] - \text{KL}[q_\phi(\mathbf{z}|\mathbf{x}) || p_\lambda(\mathbf{z})]$$

We can approximate it using **MC sample**.

$$\mathcal{L}(\mathbf{x}) \approx \frac{1}{S} \sum_{s=1}^S [\log p_\theta(\mathbf{x}|\mathbf{z}_s) - \log q_\phi(\mathbf{z}_s|\mathbf{x}) + \log p_\lambda(\mathbf{z}_s)]$$

DGM: Variational Auto-Encoder

Our objective is the evidence lower bound.

$$\log p(\mathbf{x}) \geq \mathbb{E}_{\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] - \text{KL}[q_\phi(\mathbf{z}|\mathbf{x}) || p_\lambda(\mathbf{z})]$$

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How to properly calculate gradients (*i.e.*, train the model)?

DGM: Variational Auto-Encoder

$$\mathcal{L}(\mathbf{x}) \approx \frac{1}{S} \sum_{s=1}^S [\log p_{\theta}(\mathbf{x}|\mathbf{z}_s) - \log q_{\phi}(\mathbf{z}_s|\mathbf{x}) + \log p_{\lambda}(\mathbf{z}_s)]$$

PROBLEM: calculating gradient wrt parameters of the variational posterior (*i.e.*, sampling process).

DGM: Variational Auto-Encoder

$$\mathcal{L}(\mathbf{x}) \approx \frac{1}{S} \sum_{s=1}^S [\log p_{\theta}(\mathbf{x}|\mathbf{z}_s) - \log q_{\phi}(\mathbf{z}_s|\mathbf{x}) + \log p_{\lambda}(\mathbf{z}_s)]$$

PROBLEM: calculating gradient wrt parameters of the variational posterior (*i.e.*, sampling process).

SOLUTION: use a non-centered parameterization (a.k.a. *reparameterization trick*).

$$q_{\phi}(\mathbf{z}|\mathbf{x}) = \mathcal{N}(\mathbf{z}|\mu, \sigma^2)$$
$$\mathbf{z}_s = \mu + \sigma \odot \epsilon, \quad \epsilon \sim \mathcal{N}(0, \mathbf{I})$$

DGM: Variational Auto-Encoder

$$\mathcal{L}(\mathbf{x}) \approx \frac{1}{S} \sum_{s=1}^S [\log p_{\theta}(\mathbf{x}|\mathbf{z}_s) - \log q_{\phi}(\mathbf{z}_s|\mathbf{x}) + \log p_{\lambda}(\mathbf{z}_s)]$$

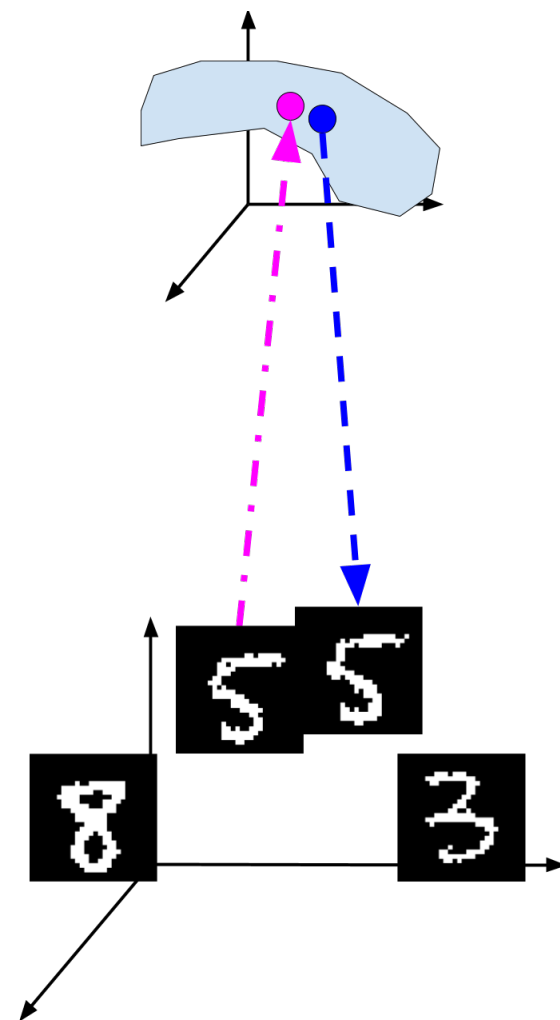
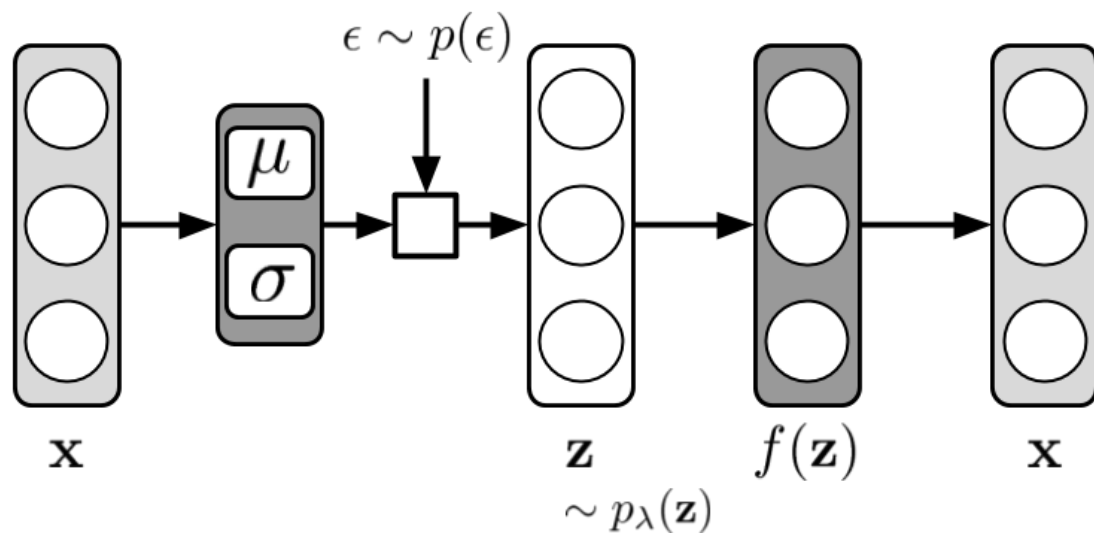
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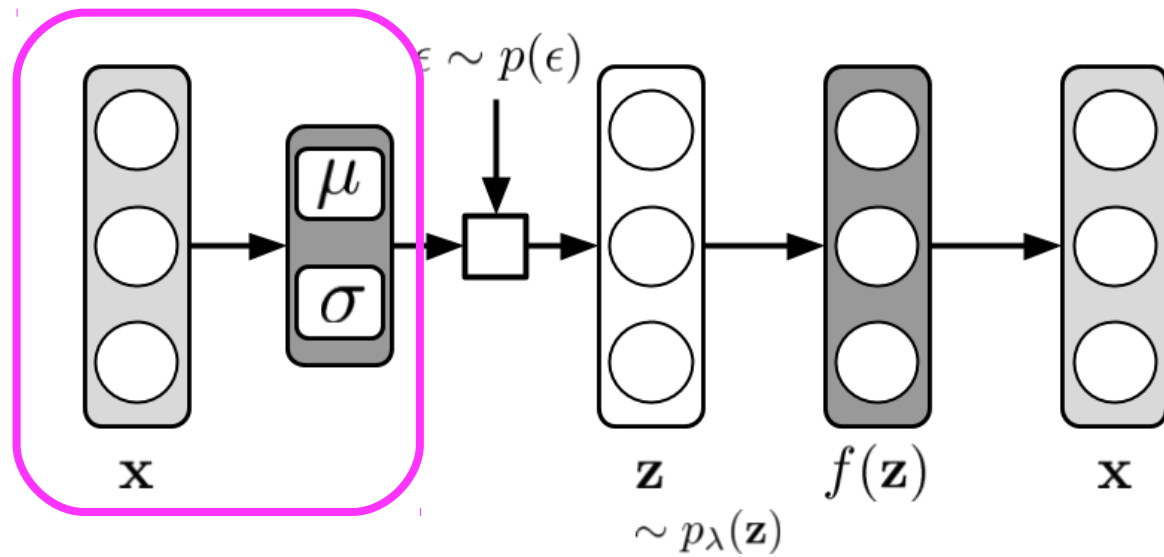
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Output of a neural network

DGM: Variational Auto-Encoder

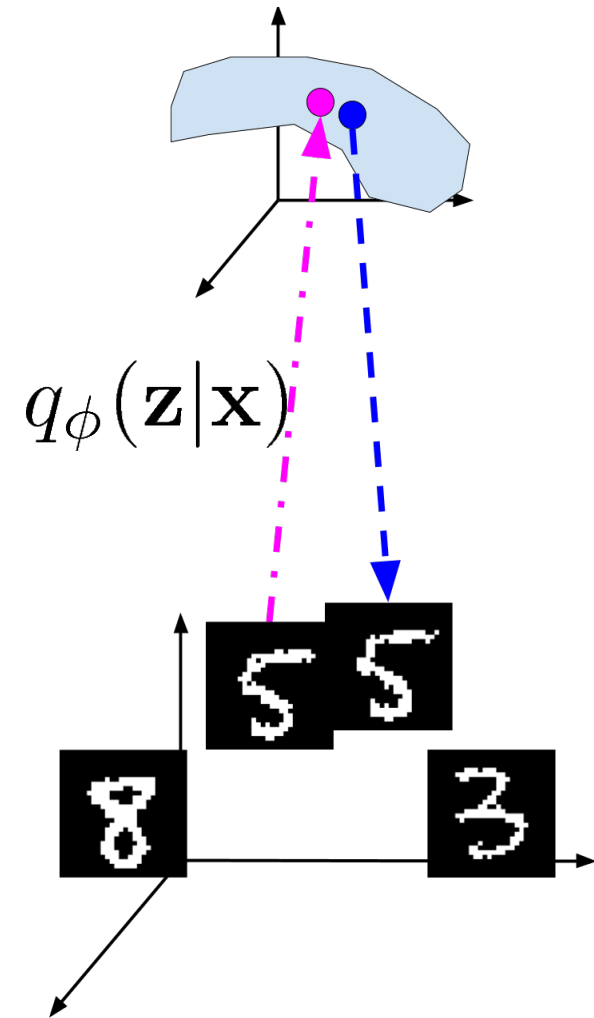


DGM: Variational Auto-Encoder

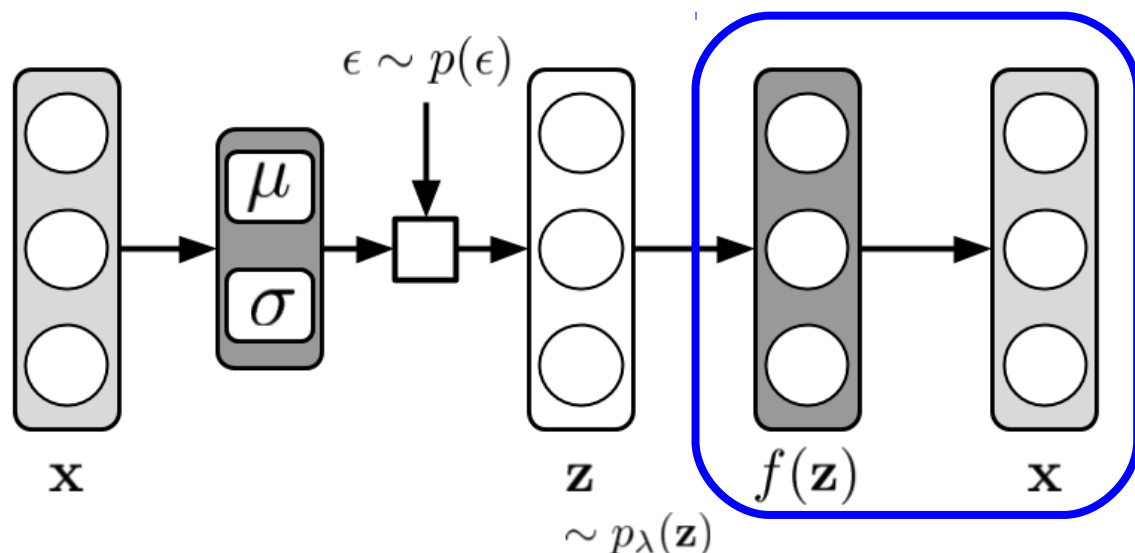


A **deep neural net** that outputs parameters of the variational posterior (**encoder**):

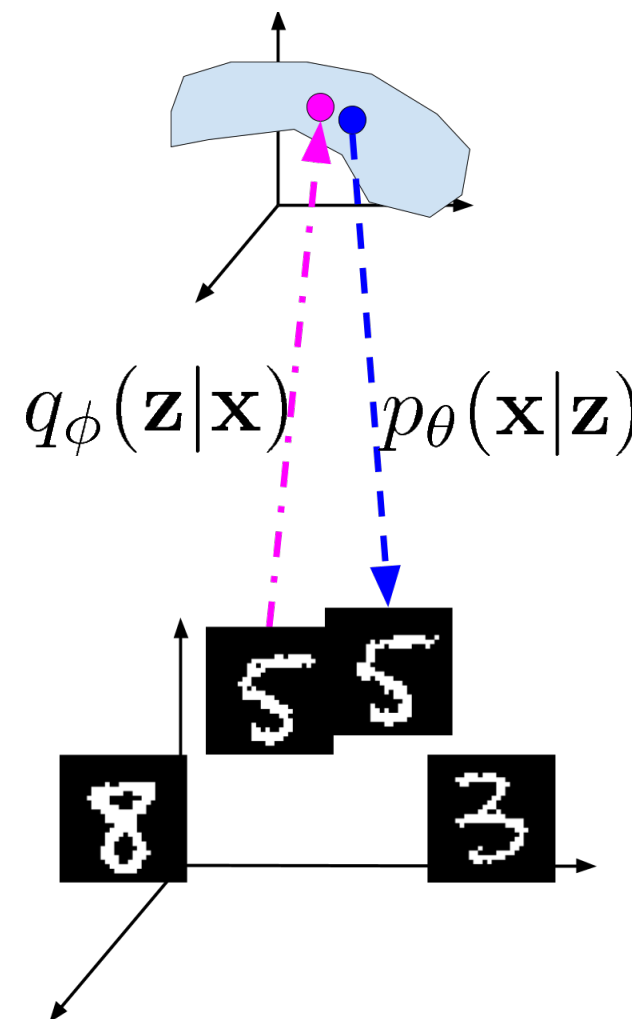
$$q_{\phi}(\mathbf{z}|\mathbf{x}) = \mathcal{N}(\mathbf{z}|\underline{\mu(\mathbf{x})}, \text{diag}\{\underline{\sigma^2(\mathbf{x})}\})$$



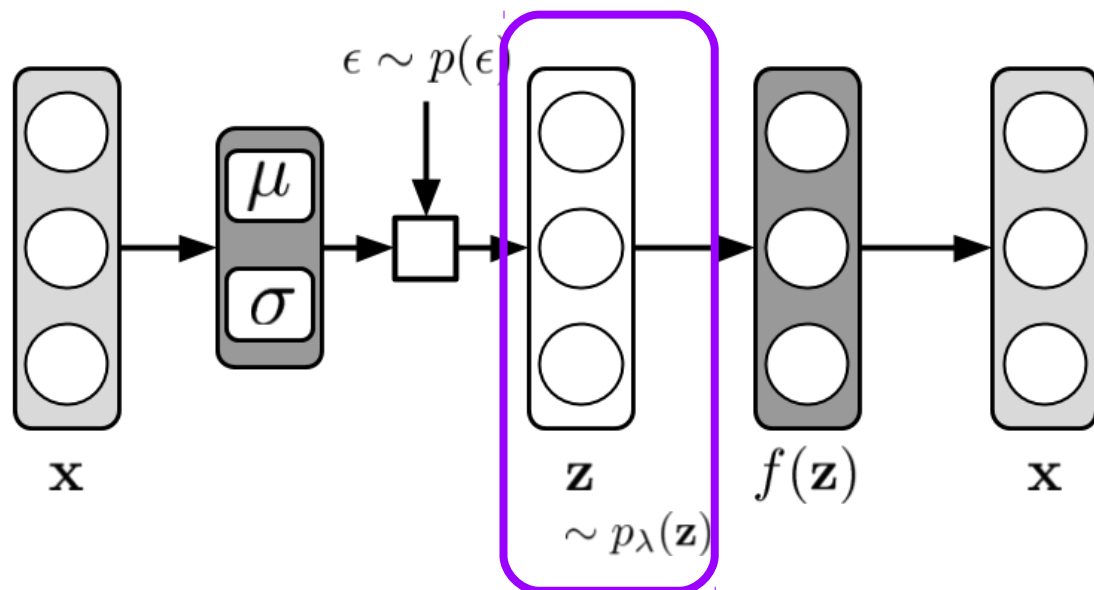
DGM: Variational Auto-Encoder



A **deep neural net** that outputs parameters of the generator (**decoder**), *e.g.*, a normal distribution or Bernoulli distribution.

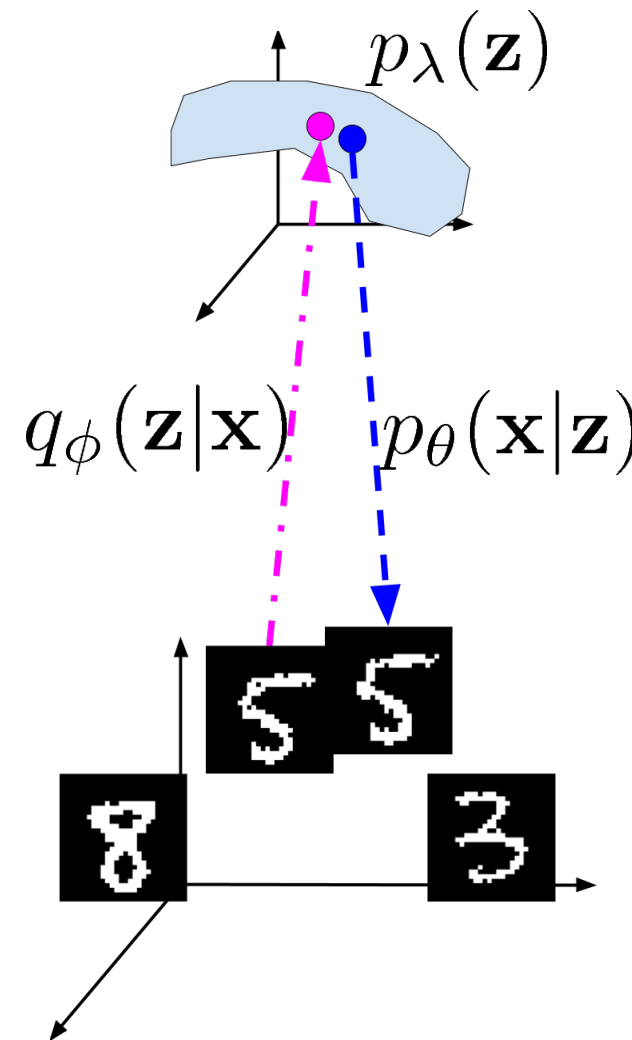


DGM: Variational Auto-Encoder



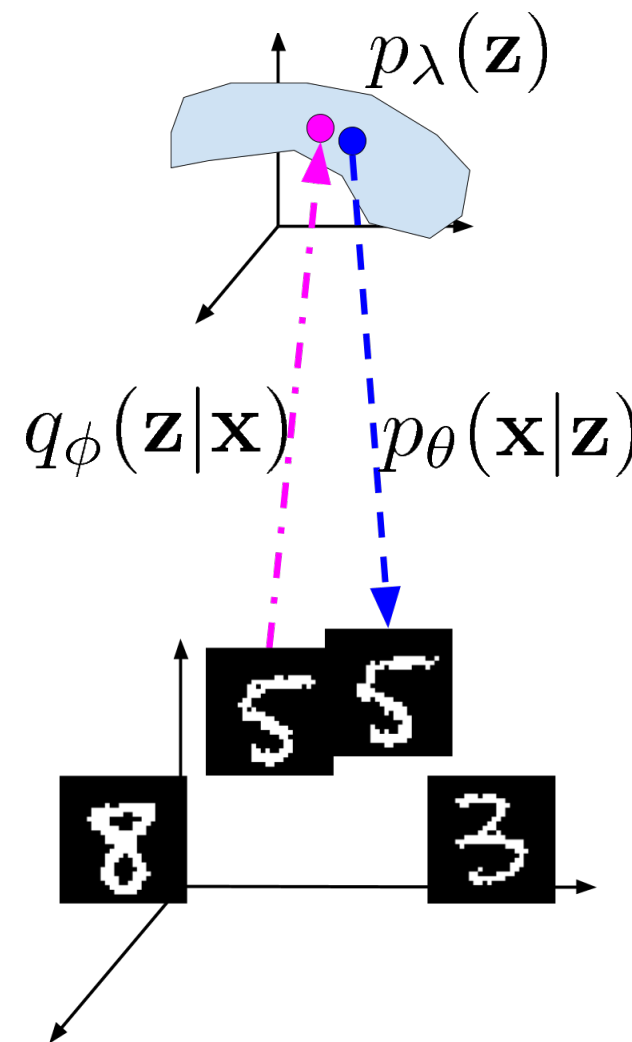
A **prior** that **regularizes** the encoder and takes part in the **generative process**.

$$p_\lambda(\mathbf{z}) = \mathcal{N}(\mathbf{z}|0, \mathbf{I})$$



DGM: Variational Auto-Encoder

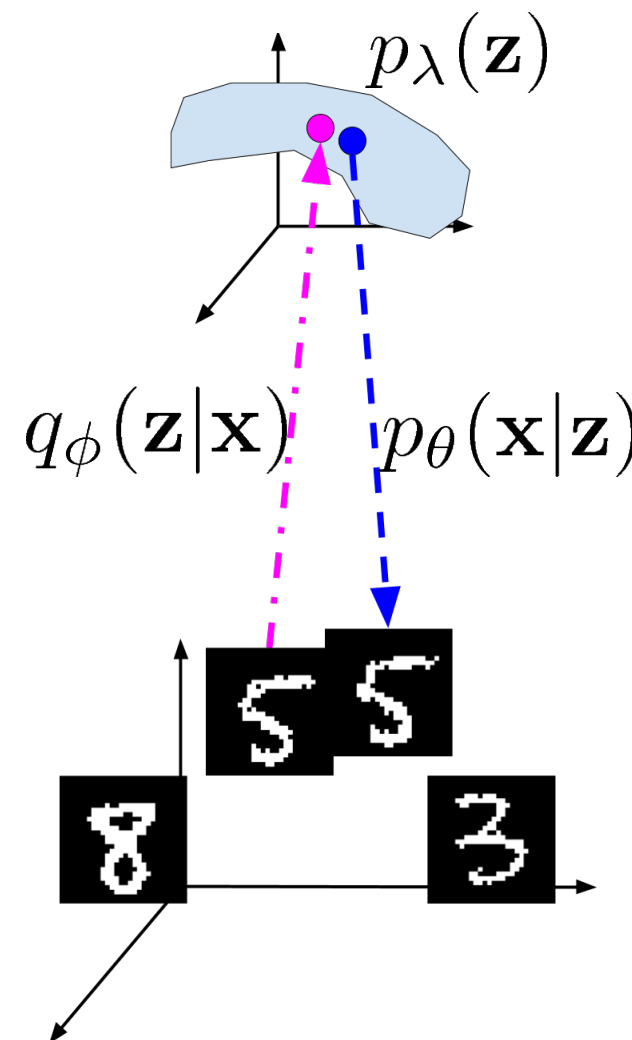
$$q_{\phi}(\mathbf{z}|\mathbf{x}) \propto p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\lambda}(\mathbf{z})$$



DGM: Variational Auto-Encoder

$$q_{\phi}(\mathbf{z}|\mathbf{x}) \propto p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\lambda}(\mathbf{z})$$

Feedforward nets
Convolutional nets
PixelCNN
Gated PixelCNN

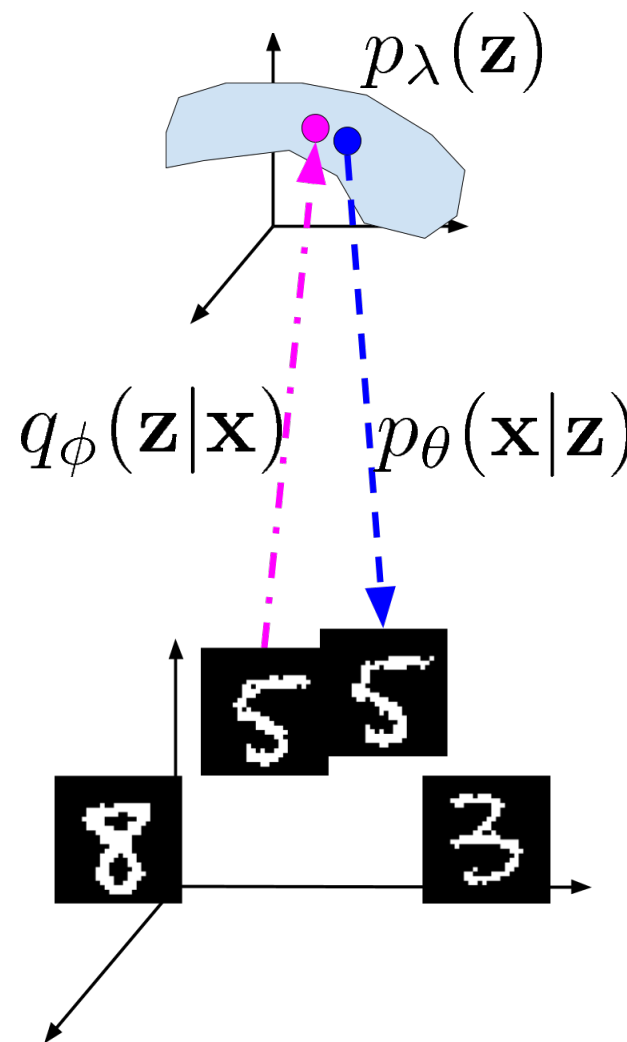


DGM: Variational Auto-Encoder

$$q_{\phi}(\mathbf{z}|\mathbf{x}) \propto p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\lambda}(\mathbf{z})$$

Normalizing flows
Volume-preserving flows
Gaussian processes
Stein Particle Descent
Operator VI

Feedforward nets
Convolutional nets
PixelCNN
Gated PixelCNN



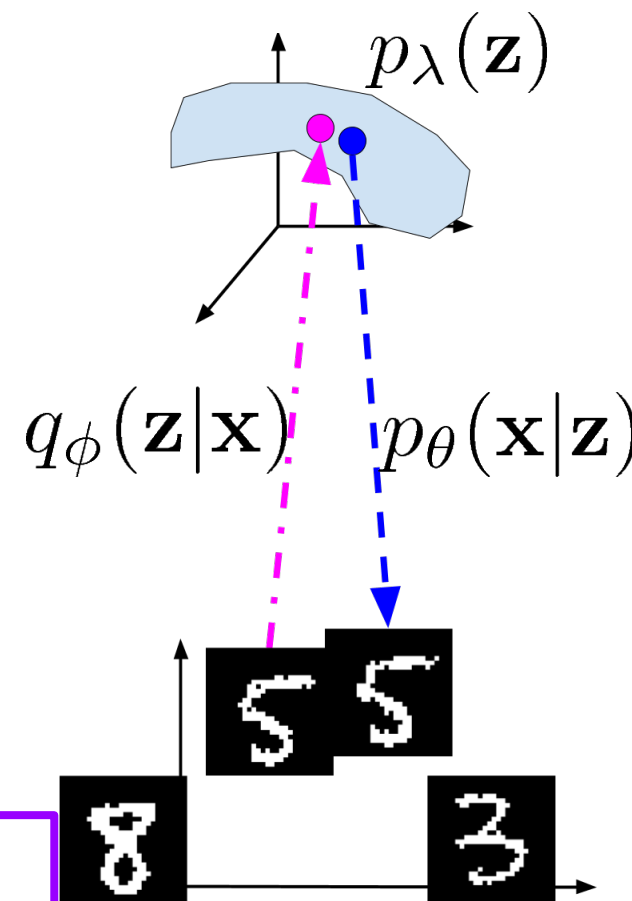
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Normalizing flows
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Feedforward nets
Convolutional nets
PixelCNN
Gated PixelCNN

Auto-regressive Prior
Objective Prior
Stick-Breaking Prior
VampPrior



DGM: Variational Auto-Encoder

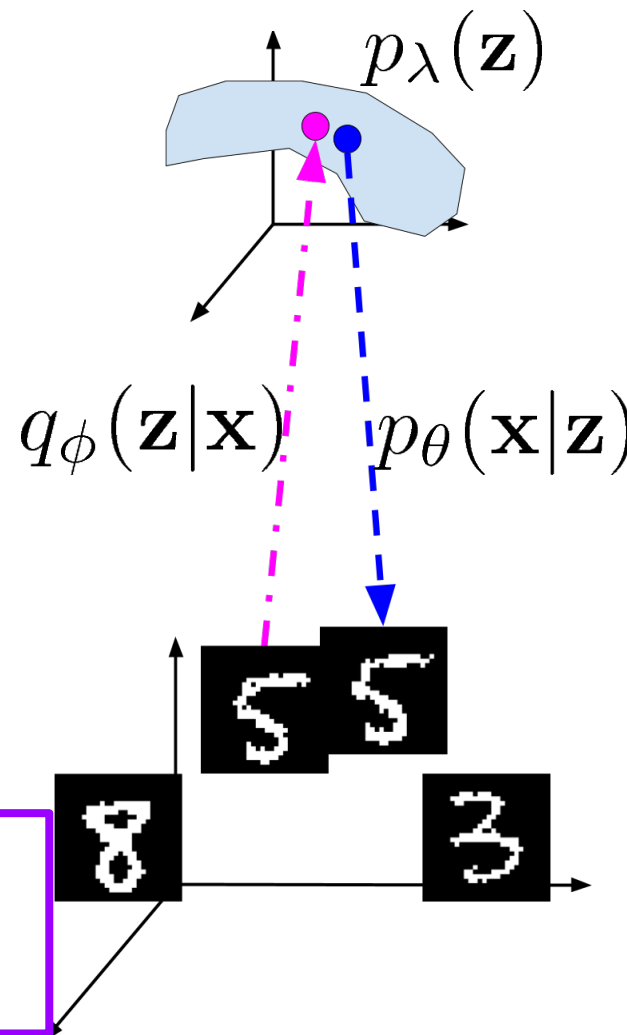
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Normalizing flows
Volume-preserving flows
Gaussian processes
Stein Particle Descent
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Feedforward nets
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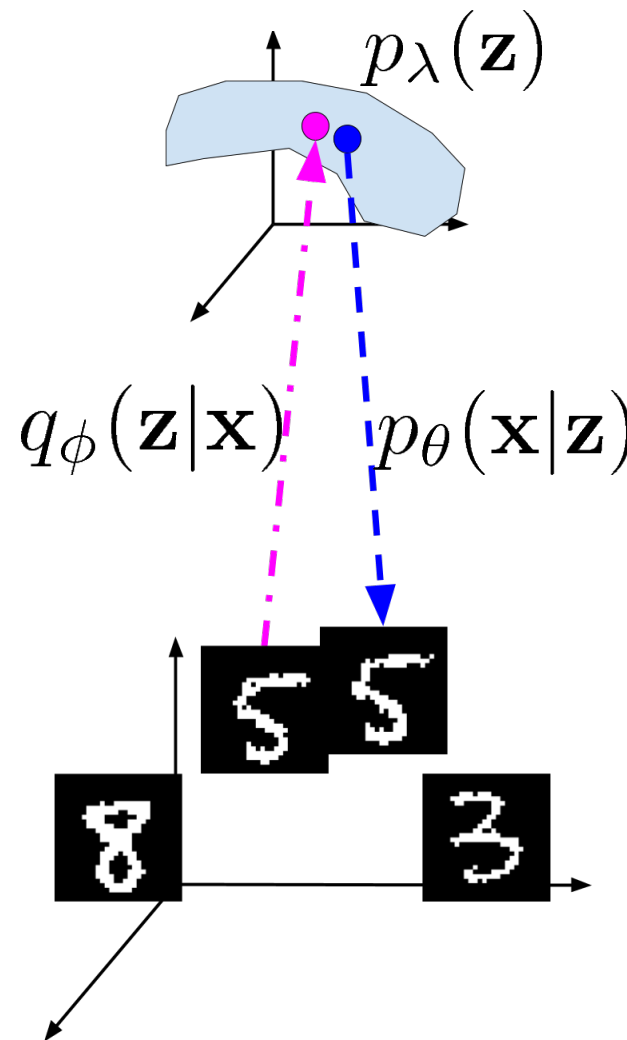
Auto-regressive Prior
Objective Prior
Stick-Breaking Prior
VampPrior

Importance Weighted AE
Renyi Divergence
Stein Divergence



Improving the posterior

$$q_{\phi}(\mathbf{z}|\mathbf{x}) \propto p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\lambda}(\mathbf{z})$$



Normalizing flows

- Diagonal posterior – **insufficient** and **inflexible**.
- How to get more flexible posterior?
→ apply a series of T invertible transformations
 $\mathbf{f}^{(t)}$ to $\mathbf{z}^{(0)} \sim q(\mathbf{z}|\mathbf{x})$.

- New objective:

$$\ln p(\mathbf{x}) \geq \mathbb{E}_{q(\mathbf{z}^{(0)}|\mathbf{x})} \left[\ln p(\mathbf{x}|\mathbf{z}^{(T)}) + \sum_{t=1}^T \ln \left| \det \frac{\partial \mathbf{f}^{(t)}}{\partial \mathbf{z}^{(t-1)}} \right| \right] - \text{KL}(q(\mathbf{z}^{(0)}|\mathbf{x}) || p(\mathbf{z}^{(T)})).$$

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$$q(\mathbf{z}^{(T)}|\mathbf{x}) = q(\mathbf{z}^{(0)}|\mathbf{x}) \prod_{t=1}^T \left| \det \frac{\partial \mathbf{f}^{(t)}}{\partial \mathbf{z}^{(t-1)}} \right|^{-1}.$$

Normalizing flows

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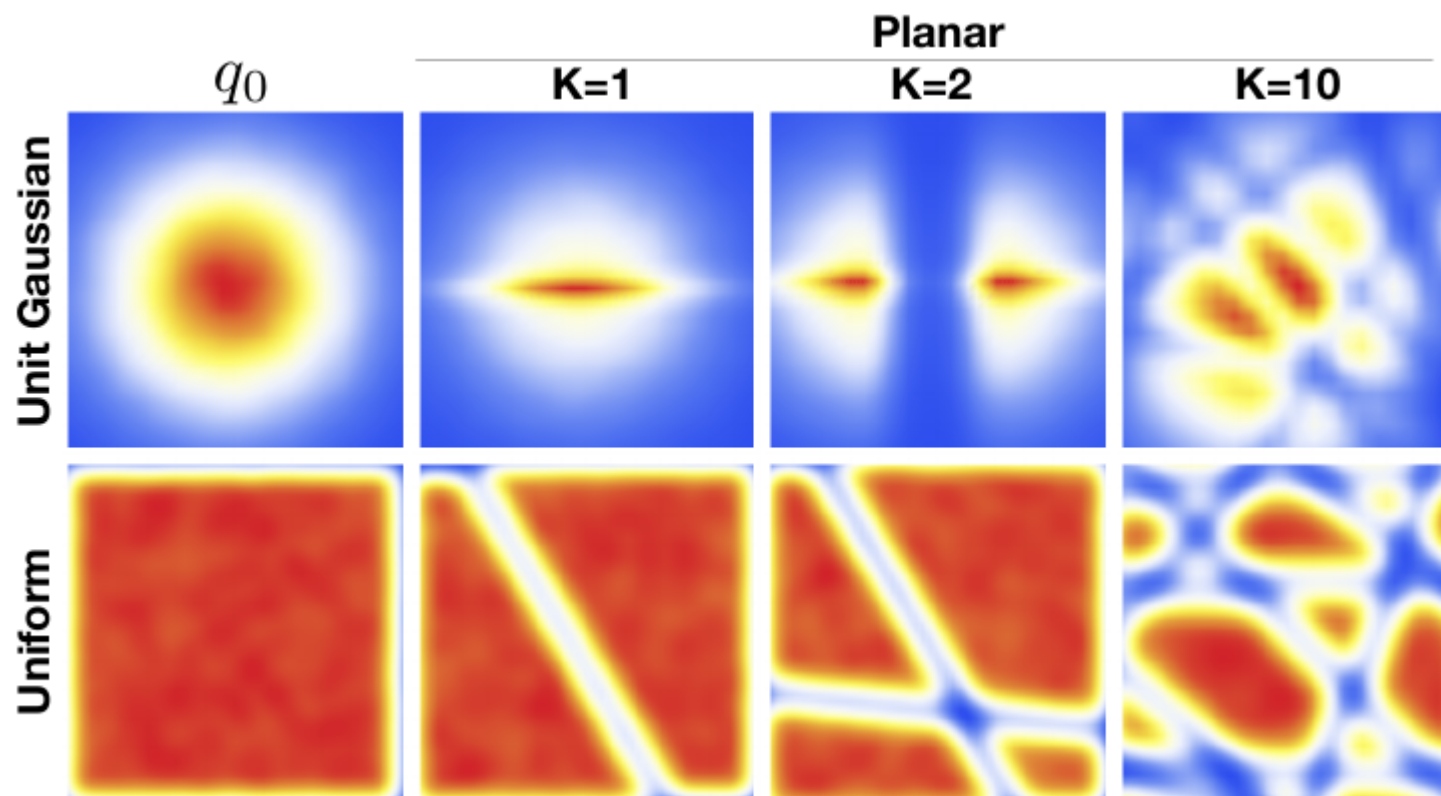
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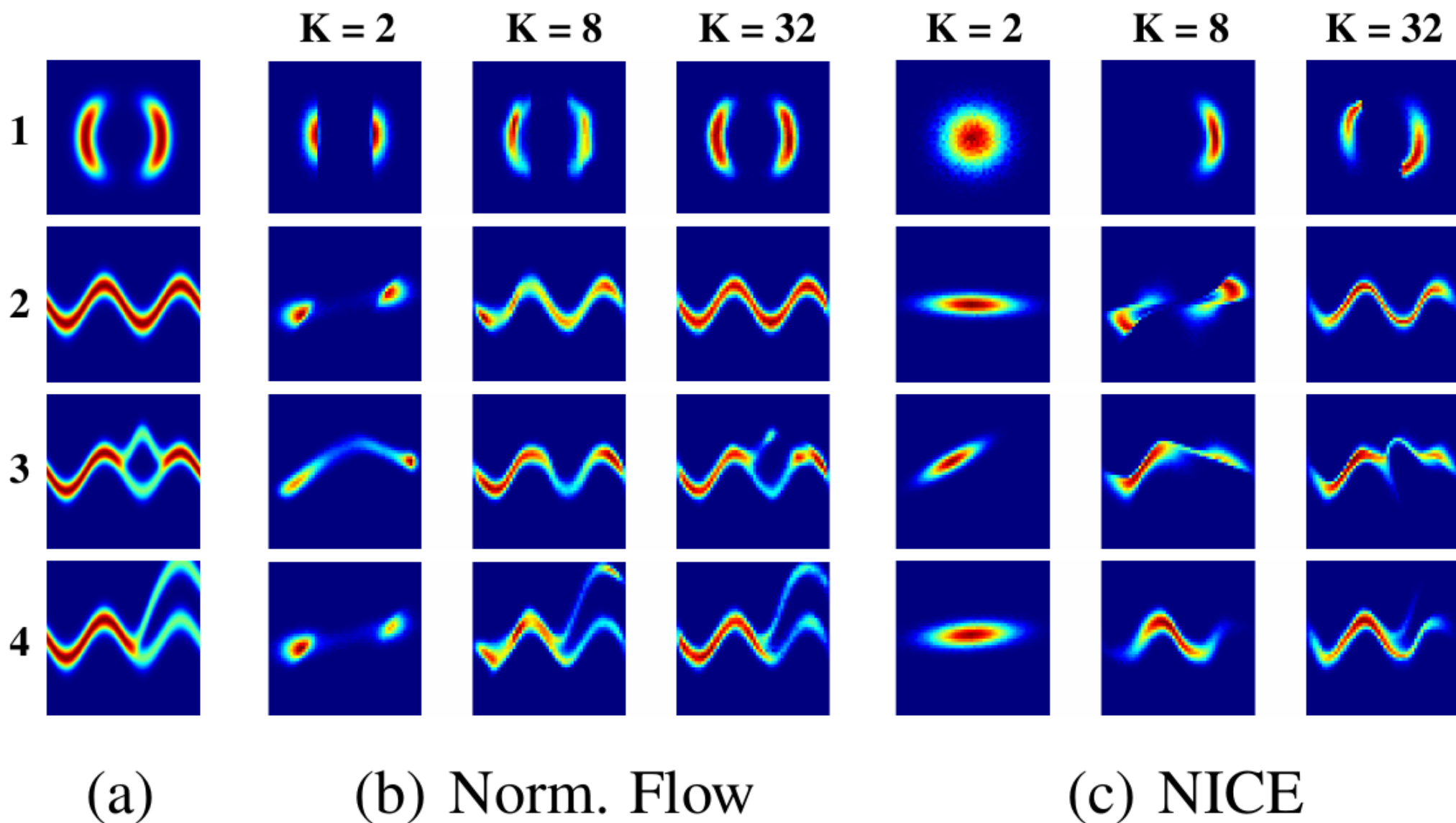
Jacobian-determinant: (i) general normalizing flow ($|\det \mathbf{J}|$ is **easy** to compute)

(ii) volume-preserving flow, *i.e.*, $|\det \mathbf{J}|=1$

Normalizing Flow



Normalizing Flow



Extensions of normalizing flows

- How to obtain more **flexible** posterior and preserve $|\det J|=1$?
 - using *orthogonal matrices* $\rightarrow$ **Householder flow**

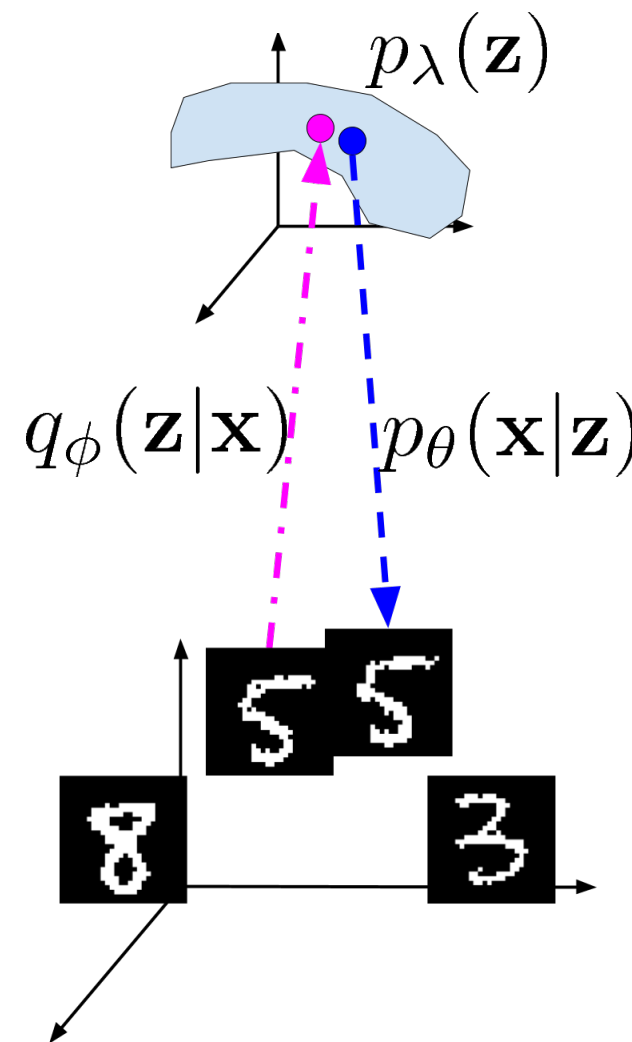
Tomczak, J. M., & Welling, M. (2016). Improving Variational Inference with Householder Flow. arXiv preprint arXiv:1611.09630. NIPS Workshop on Bayesian Deep Learning 2016

- **General** normalizing flow:
 - using *autoregressive model* $\rightarrow$ **Inverse Autoregressive Flow**

Kingma, D. P., Salimans, T., Jozefowicz, R., Chen, X., Sutskever, I., & Welling, M. (2016). Improving Variational Inference with Inverse Autoregressive Flow. NIPS 2016

Improving the decoder

$$q_{\phi}(\mathbf{z}|\mathbf{x}) \propto \underbrace{p_{\theta}(\mathbf{x}|\mathbf{z})}_{\text{decoder}} p_{\lambda}(\mathbf{z})$$

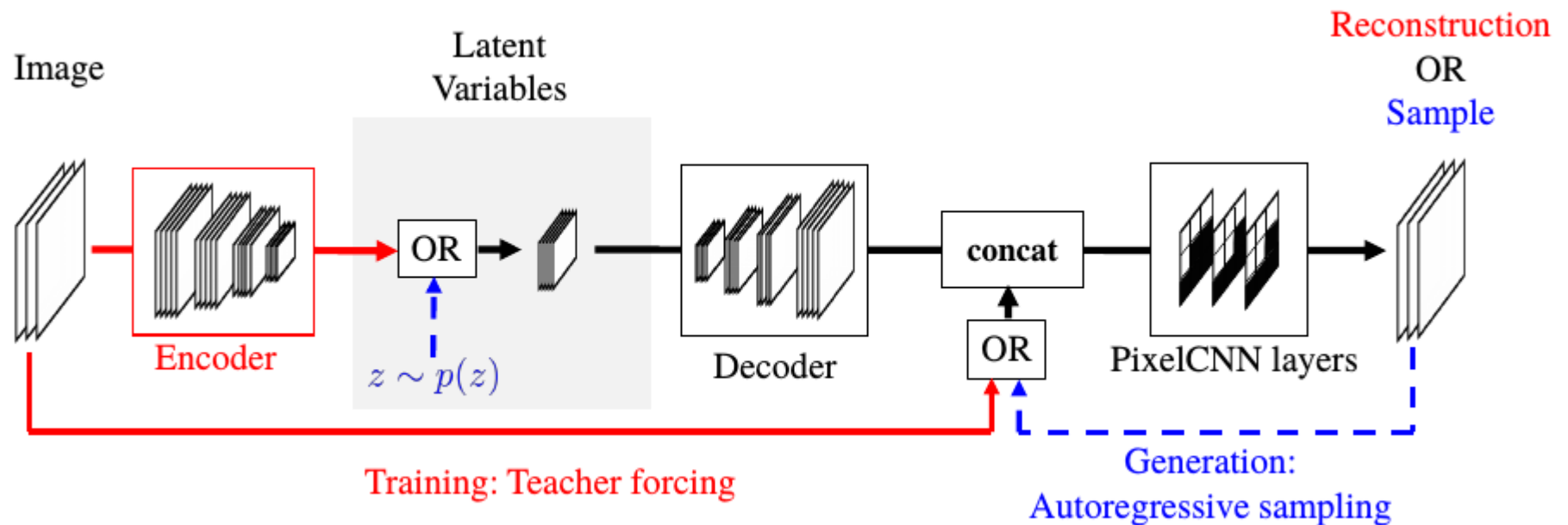


Improving the decoder

- Dependency only on $\mathbf{z}$ – **missing correlations**.
- How to get more flexible decoderposterior?
→ apply **autoregressive model**

$$p(\mathbf{x}|\mathbf{z}) = p(x_1|\mathbf{z}) \prod_{d=2}^D p(x_d|x_1, \dots, x_{d-1}, \mathbf{z})$$

PixelVAE (PixelCNN + VAE)



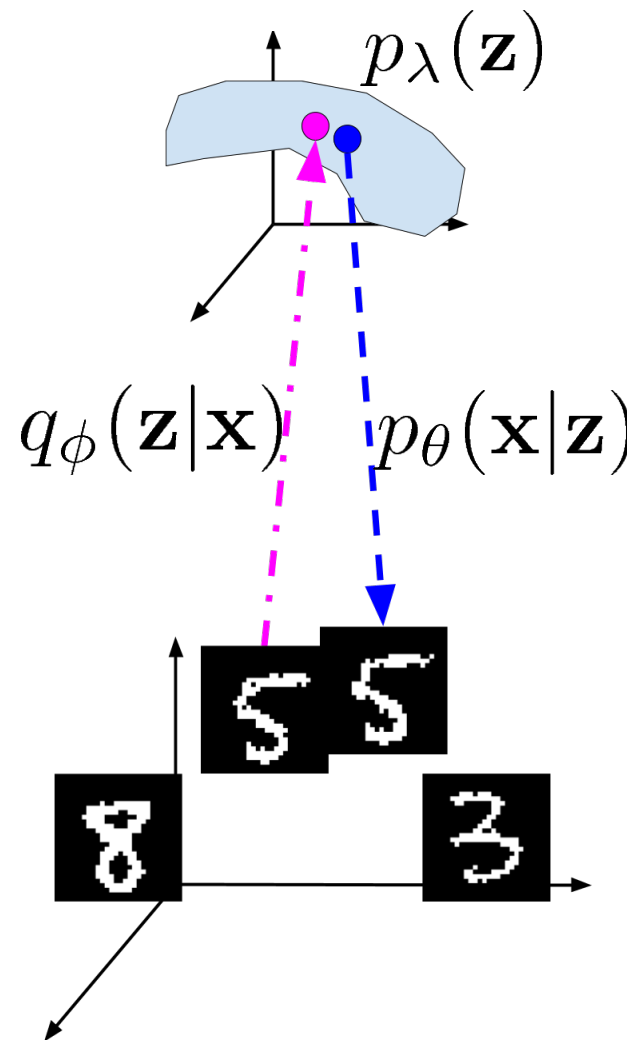
PixelVAE (PixelCNN + VAE)



Figure 6: Samples from hierarchical PixelVAE on the 64x64 ImageNet dataset.

Improving the prior

$$q_{\phi}(\mathbf{z}|\mathbf{x}) \propto p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\lambda}(\mathbf{z})$$



Improving the prior

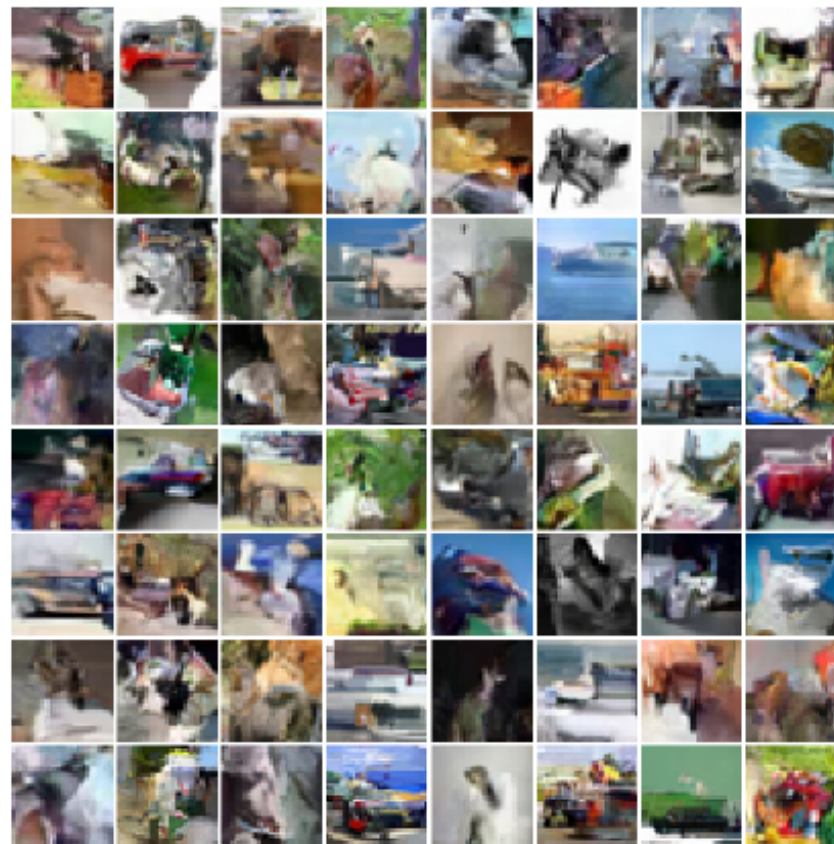
- Standard normal prior – **unimodal, too restrictive.**
- How to get more flexible prior?
→ apply **autoregressive prior**

Chen, X., Kingma, D. P., Salimans, T., Duan, Y., Dhariwal, P., Schulman, J., ... & Abbeel, P. (2016). Variational lossy autoencoder. arXiv preprint arXiv:1611.02731.

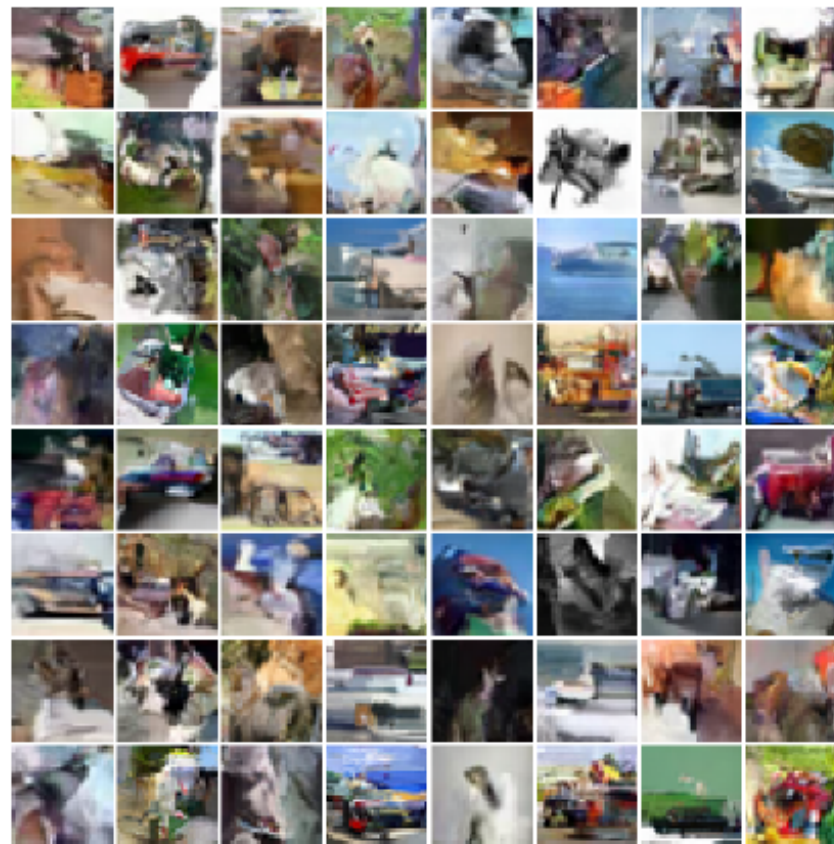
→ apply **variational mixture of posteriors (VampPrior)**

Tomczak, J. M., & Welling, M. (2017). VAE with a VampPrior. arXiv preprint arXiv:1705.07120.

Autoregressive prior



Autoregressive prior



VampPrior



(a) real data

(b) VAE

(c) HVAE + VampPrior

(d) convHVAE + VampPrior

Some extensions and applications of VAE

- Semi-supervised learning with VAE.

Kingma, D. P., Mohamed, S., Rezende, D. J., & Welling, M. (2014). *Semi-supervised learning with deep generative models*. NIPS

- VAE for sequences.

Bowman, S. R., Vilnis, L., Vinyals, O., Dai, A. M., Jozefowicz, R., & Bengio, S. (2015). *Generating sentences from a continuous space*. arXiv preprint arXiv:1511.06349.

Chung, J., Kastner, K., Dinh, L., Goel, K., Courville, A. C., & Bengio, Y. (2015). *A recurrent latent variable model for sequential data*. NIPS

- More powerful decoders (using PixelCNN).

Gulrajani, I., Kumar, K., Ahmed, F., Taiga, A. A., Visin, F., Vazquez, D., & Courville, A. (2016). *PixelVAE: A latent variable model for natural images*. arXiv preprint arXiv:1611.05013.

Chen, X., Kingma, D. P., Salimans, T., Duan, Y., Dhariwal, P., Schulman, J., ... & Abbeel, P. (2016). *Variational lossy autoencoder*. arXiv preprint arXiv:1611.02731.

Some extensions and applications of VAE

- Applications: graph data

Kipf, T. N., & Welling, M. (2016). *Variational Graph Auto-Encoders*. arXiv preprint arXiv:1611.07308. NIPS Workshop

Berg, R. V. D., Kipf, T. N., & Welling, M. (2017). *Graph Convolutional Matrix Completion*. arXiv preprint arXiv:1706.02263.

- Applications: drug response prediction

Rampasek, L., & Goldenberg, A. (2017). *Dr.VAE: Drug Response Variational Autoencoder*. arXiv preprint arXiv:1706.08203.

- Applications: text generation

Yang, Z., Hu, Z., Salakhutdinov, R., & Berg-Kirkpatrick, T. (2017). *Improved Variational Autoencoders for Text Modeling using Dilated Convolutions*. arXiv preprint arXiv:1702.08139.

DGM: VAE

PROS

Log-likelihood framework

Easy **sampling**

Training using **gradient-based methods**

Stable training

Discovers **latent representation**

Could be easily combined with other probabilistic frameworks

CONS

Only **explicit** models

Produces **blurry images(?)**

1283 + 1146

Number of citations of seminal papers on
GANs and VAE.*

**According to GoogleScholar, 26.09.2017*

In order to *make better
decisions*, we need a
*better understanding
of **reality***.

=

generative modeling

Web-page:

<https://jmtomczak.github.io>

Code on github:

<https://github.com/jmtomczak>

Contact:

J.M.Tomczak@uva.nl

jakubmkt@gmail.com

Part of the presented research was funded by the European Commission within the Marie Skłodowska-Curie Individual Fellowship (Grant No. 702666, "*Deep learning and Bayesian inference for medical imaging*").



RESEARCH & INNOVATION
Marie Skłodowska-Curie actions